A digital-analogue match filter for piecewise square pulses.

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It is well known that the optimum signal to noise ratio in a radar is obtained when the receiver response is matched to the transmitter pulse shape. A matched receiver has an impulse response, $g(t)$, equal to the negative fold of the transmitter pulse shape $p(t)$, i.e.:

$$g(t) = p(-t) \tag{1}$$

On the other hand we have that for a given maximum peak and average power the pulse length is shortest when the transmitter pulse has a square or piecewise square shape (sequences of positive and negative pulses). In many cases the square shapes are the easiest, if not the only posible, shapes that can be obtained in a transmitter. Here we use "square" to imply pulses with fairly constant amplitude and relative short transit times.

Square-wave receiver input responses can not be obtained with simple lumped element filters. One needs either tapped delay lines or digital techniques. The present note describes a hybrid analogue-digital technique to achieve a matched set of piecewise "square" transmitter pulse shapes and receiver input response. The scheme works also for phase $(0-180^{\circ})$ coded pulse sequences. The scheme is a natural use for a digital decoder for phase coded pulses.

First we shall describe the operation for a single square pulse. The extension to a phase coded sequence is straightforward..

Given a sequence of N narrow input pulses of equal spacing,

$$\sum_{i=o}^{N-1} \delta(t-ci) \qquad 2$$

where c is the separation between pulses, we can produce a square pulse by convolving it with a pulse function, h(t) of a width comparable to the inpulse separation a. Figure 1, respents a "square" wave obtained by convolving the inpulse sequence by a bell shaped function as the one depicted in dotted lines.

In practice, a transmitter impulse of similar shape can be implemented by passing a sequence of narrow inpulses through a pulse filter of width c. If we represent the narrow pulses by delta functions - a valid aproximation when the width of the pulses is much less than c - we can represent the transmitter pulse shape by

$$p(t) = \int dt' \sum_{i=1}^{N} \delta(t'-ic)\, h_e(t-t') \qquad 3$$

of which the function plotted in figure 1 is an example. We can implement a receiver with the corresponding matched inpulse response by an analogue filter with input response $f(t) = h_e(-t)$ and a digital integrator programed to add N consecutive samples at intervals c. The whole transmitter and receiver scheme is depected in figure 2. The ADC produces a sequence of sample values S_1, S_2 ...S at times t_h+c, t_h+2c, ..., t_h+nc and adds them all two produce a sample value of $r_o(t)$ corresponding to time t_n.

Let $r_f(t)$ be the output of the filter and $r_2(t)$ the signal received then

$$r_f(t) = \int dt' \; r_i(t')f(t-t') = \int dt' \; r_i(t')h(t'-t)$$

where we have made use of the fact that $f(t)=h(-t)$ (in reallty only a good aproximation since real pulse filters (or equivalent) do not produce perfect symmetrical shapes. We are also ignoring the unavoidable delay in a pulse filter to simplify notation).
The sequence of samples $s(t_h+ic)$ can be represented by

$$r_f(t_h+ic) = \int dt'' \; \delta(t''-ic) \; r_f(''+t_h) \qquad 6$$

and the integrated output $r_o(t_h)$

$$r_o(t_h) = \int dt''dt' \; \sum_i^N \delta(t''-ic)r_i(t'+t_h)h(t'-t'') \qquad 7$$

or

$$r_o(t_h) = \int dt' \; r_i(t'+t_h) \; p(t) \qquad 8$$

which is the output of a matched system with input response $g(t)=p(-t)$, as desired.

By chossing consecutive sequences with initial sample time

$$t_h = ch \qquad h = 0,1,2 \; ..... \; M$$

we generate a discrete sample sequence $r_o(t_h)$ corresponding to the echoes from different altitudes.

It is not necessary to obtain all possible values of $r_o(t_h)$. Complete independent samples are obtained after a delay cN. A good practice, unless there are processing speed limitations, is to obtain a value every cN/2.

Phase coded sequences

Phase coded sequences are obtained in identical fashion with the only difference that different sign $p_i = \pm 1$ are assigned to the different narrow pulses (delta functions). The transmitter pulse then is given by

$$p(t) = \int dt' \sum_{i=1}^{N} p_i \delta(t' - ic) \, h_e \, (t - t')$$

And the decoding is performed by adding or subtracting the corresponding sample pulse $s(t_h + ic)$ according to the sign p_i corresponding to the index i. The filter and decoded signal is given by

$$r_o(t_h) = \sum_{i=1}^{N} p_i r_f(t_h + ic)$$

where $r_f(t)$ is defined as before (Eq 5 and 6). Fig. 2 shows an example of a phase coded sequence and figure 8. one possible way of obtaining the proper transmitter pulse shape.

Advantages of the system

By changing N and the sequency p_i one can synthesize arbitrary square pulse sequences of variable length either plane or coded. The T_x pulse and the corresponding matched filter can be obtained digitally under radar controller and decoder control without having to change physical bandwidth. Thus different observing schemes can be obtained by software operator control. The matching achieved is always very close to the ideal.

Selection of parameters for the SOUSY-Radar

A defining parameter for the design of the match-filter-decoder scheme is the sampling rate. In the SOUSY-Radar the maximum sampling rate is 0,5 usec determined by the desired maximum resolution, transmitter bandwidth the state of the art in analogue to digital converters and digital processing equipment.

Having selected the sampling rate, the parameter c is determined. The characteristic width of the impulse response functions h(t) and f(t) is of the same order of magnitude. The precise value for it has been selected as the minimum width which would reduce the ripple on a synthetized long square function to less than 5 %. Wider functions would produce to low of a peak value in the narrow pulses of a phase coded sequence, i.e. The filter then would not responde quick enough to a change of sign. Values selected according to this criteria also meet Nyquist criteria for the sampling of the receiver output without redundancy.

One of the desired characteristics of the functions h(t) and f(t) is their symmetry. There are several pulse filters which meet this conditions: Gaussian, Maximally flatt (Bessel), Equiripple etc. The condition of symmetry is equally well satisfied by any of them for a large number of poles, but it is found than for a given reduced number of poles, filters designed for a linear phase with equiripple error give the best symmetry. A filter with a equiripple error of 0,5 % and 6 poles has a response symmetric to a few percent. Smaller ripple error does not improve the symmetry much; the 0,5 % giving slightly faster roll off. Higher order filters increase in complexity (number of reactive elements equal to the number of poles) and do not improve much the response which already has a satisfactory shape at n=6. The fig. 1 and 3 has been obtained with such a filter with a characteristic time $1/\Omega$. = 0.25 usec and a low pass bandwidth of $f_{3db} = 2\pi\Omega$. = 0.637 Mhz. The corresponding performance curves are depicted in figures 4,5,6. The table in figure 7 gives the corresponding values for the lumped constant elements in a passive filter. Active filters can also be design having the advantage that can be synthetized using only resistive and capacitive elements.

With a sampling period of 0,5 usec. and above filter one can generate pulses form less than 0,5 usec. equivalent width to any desired length limited only by the practicallity of the decoding operation in the existing decoder, 30 microsecond pulses being way inside the range. Coded sequences can also have different Baud length although it should be mention than high average powers for a given pulse length are achieved only for Baud lengths of 1 usecond or longer. A 0,5 usec. Baud length is also possible, but with a deteriorated average power with respect to a corresponding ideal square shape. A ideal square shape of 0,5 usec. Baud length on the other hand would require a wider transmitter and receiver bandwidth and a higher sampling rate regardless of the aproach taken.

Recomended accion to implement the digital-analogue match filter in the SOUSY-Radar

Principle of operation - See accompaning notes

Radar controller - Generate the sequence of narrow pulses ba gating narrow clock pulses at a 0,5 usec. period with the T_xON pulse..

T_x-Exitation - With the narrow pulses form above and the pulse sign sequence - also from radar controller - generate the sequence of balanced positive and narrow pulses.

Construct a "pulse-filter"* for the transmitter to shape above pulses in accordance with the spedifications given below.

The output of the filter should control the low level exitation of the transmitter trough a balance mixer. See figure 8.

Receiver - Adjust IF and phase-detector bandwith to conform with specifications below.. The digital addition is performed by the decoder without modification.

Filter Specifications

- Type - Linear-Phase Equiripple $0,5^o$
- Phase error of Equiripple = $0,5^o$
- Number of poles-6 minimum and sufficient
- Frequency - See fig. 5 f_{3db}=0.637MHz
- Width of impulse response-0.556 usec between half amplitud points
- Reference -

Normalied input response of the filter - see fig. 1

*) "pulse-filter" is used here to mean bell-shaped symmetric response filter. The best symmetry is actually achived with a Linear-Phase Equiripple filter (phase error $0,5^o$)

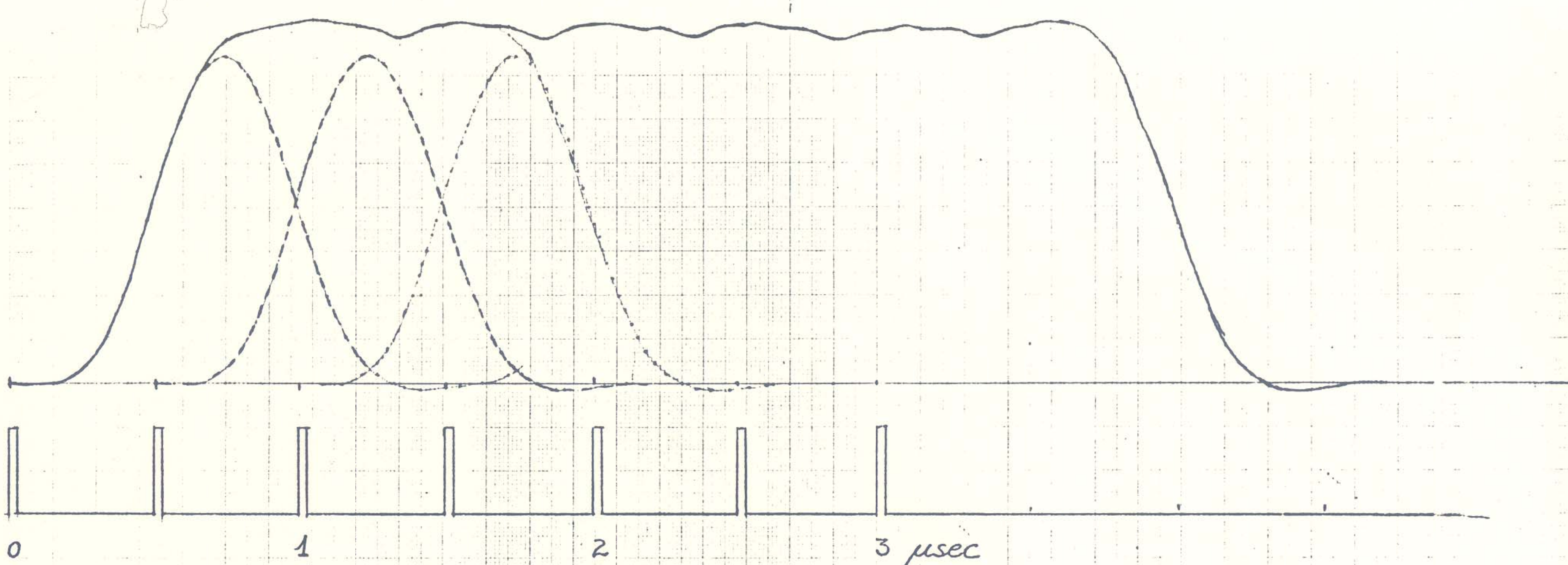


Fig1

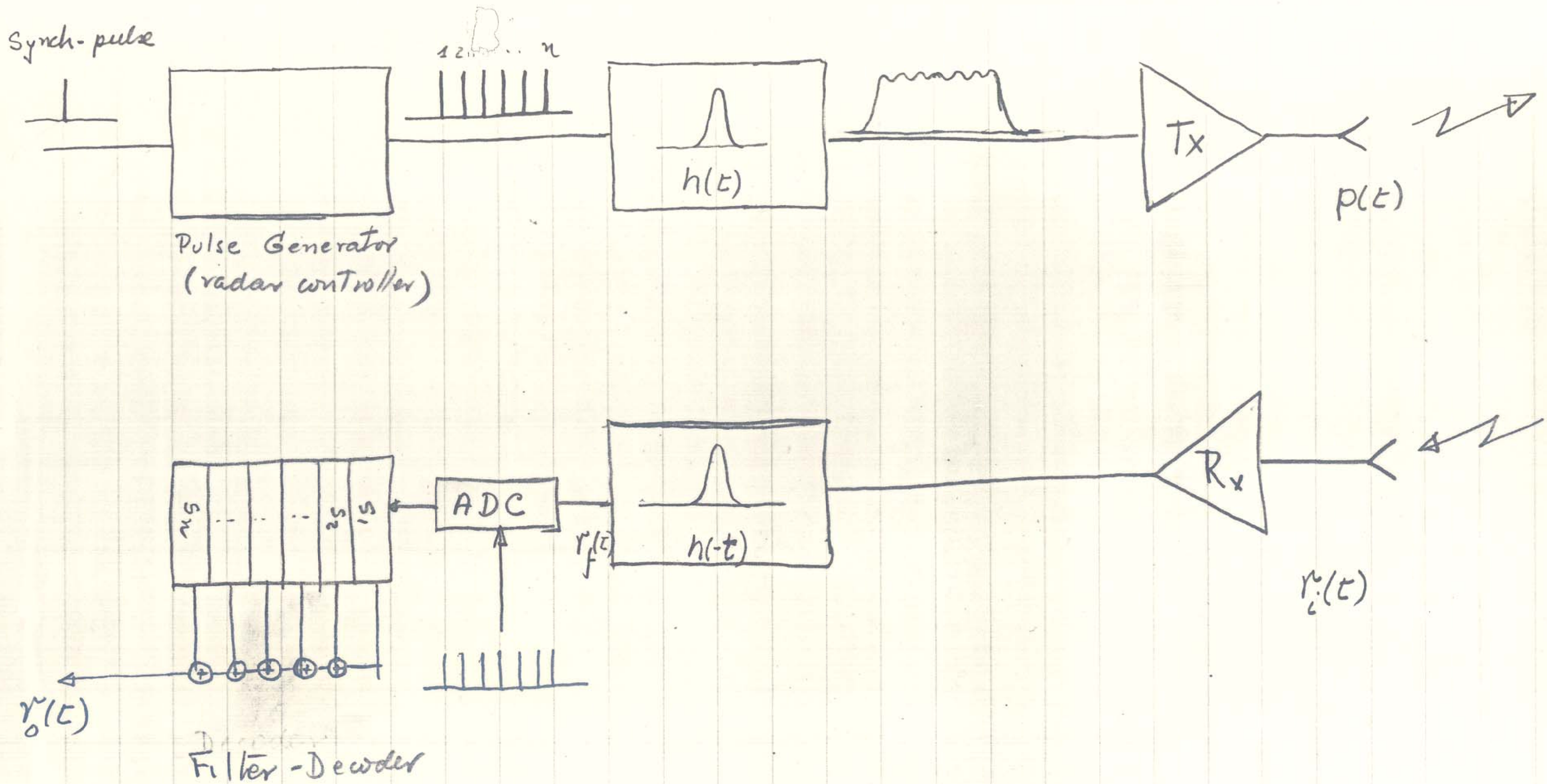


Fig 2 - Schematic diagram of a digital-analogue match radar. Phase-coded pulses are coded and decoded by changing the sign of the impulses and the sign of the additions in the decoder.

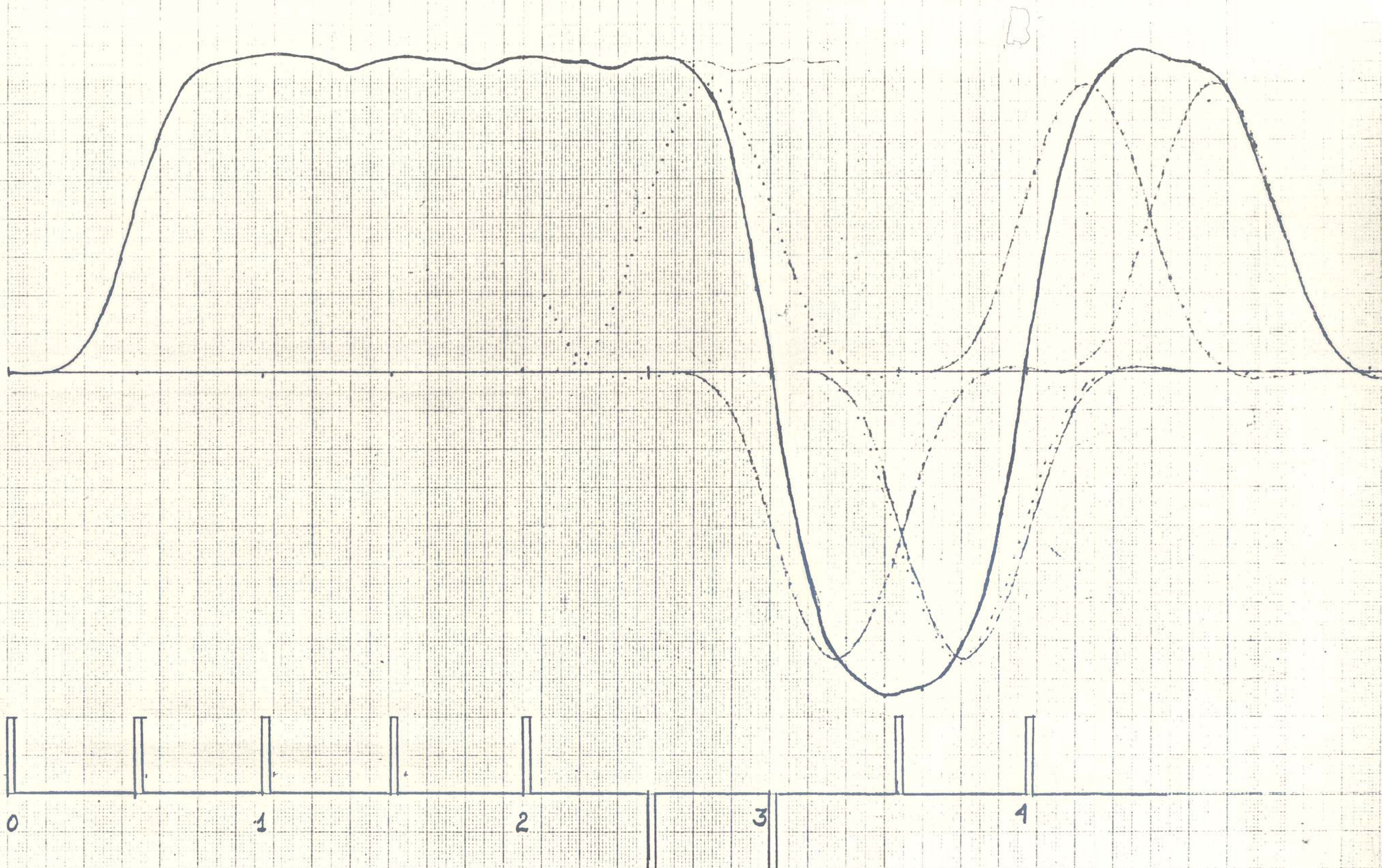
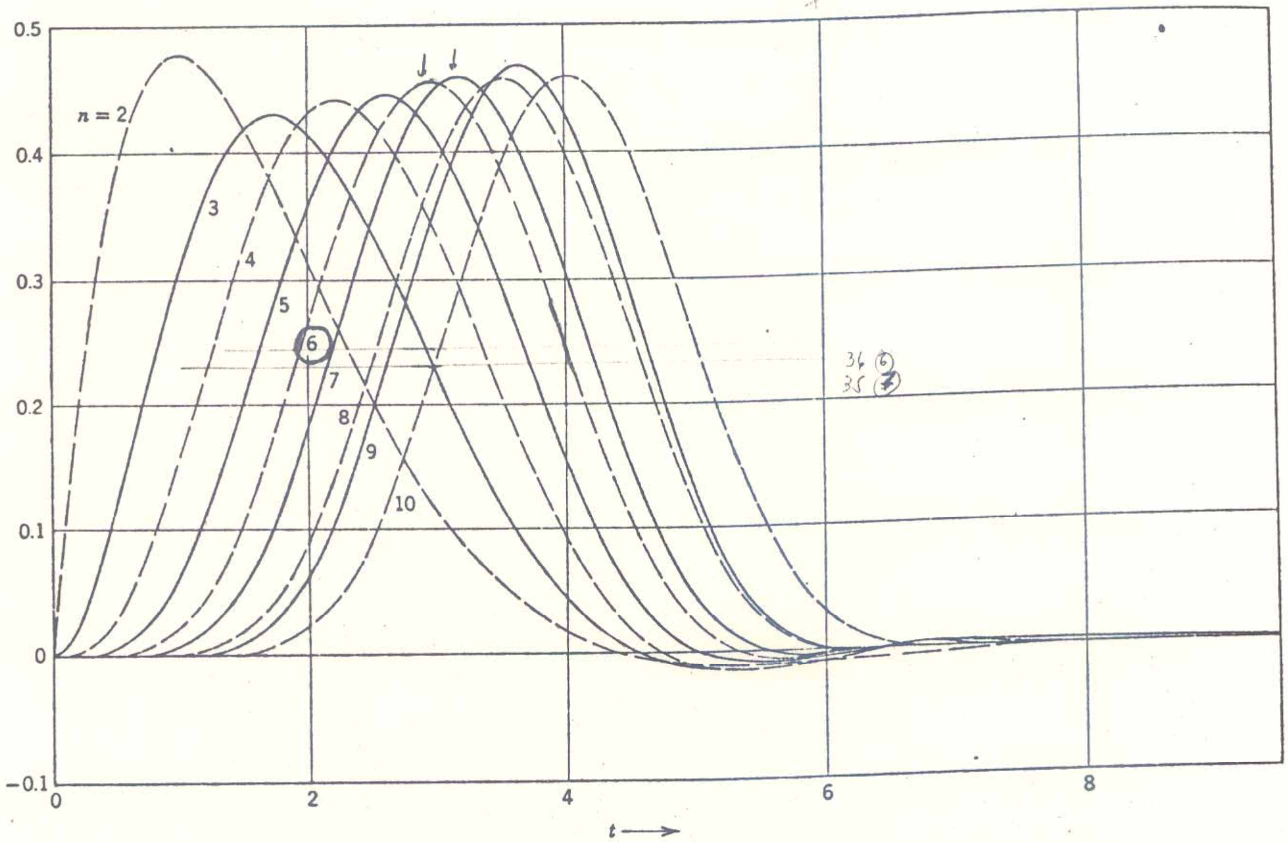
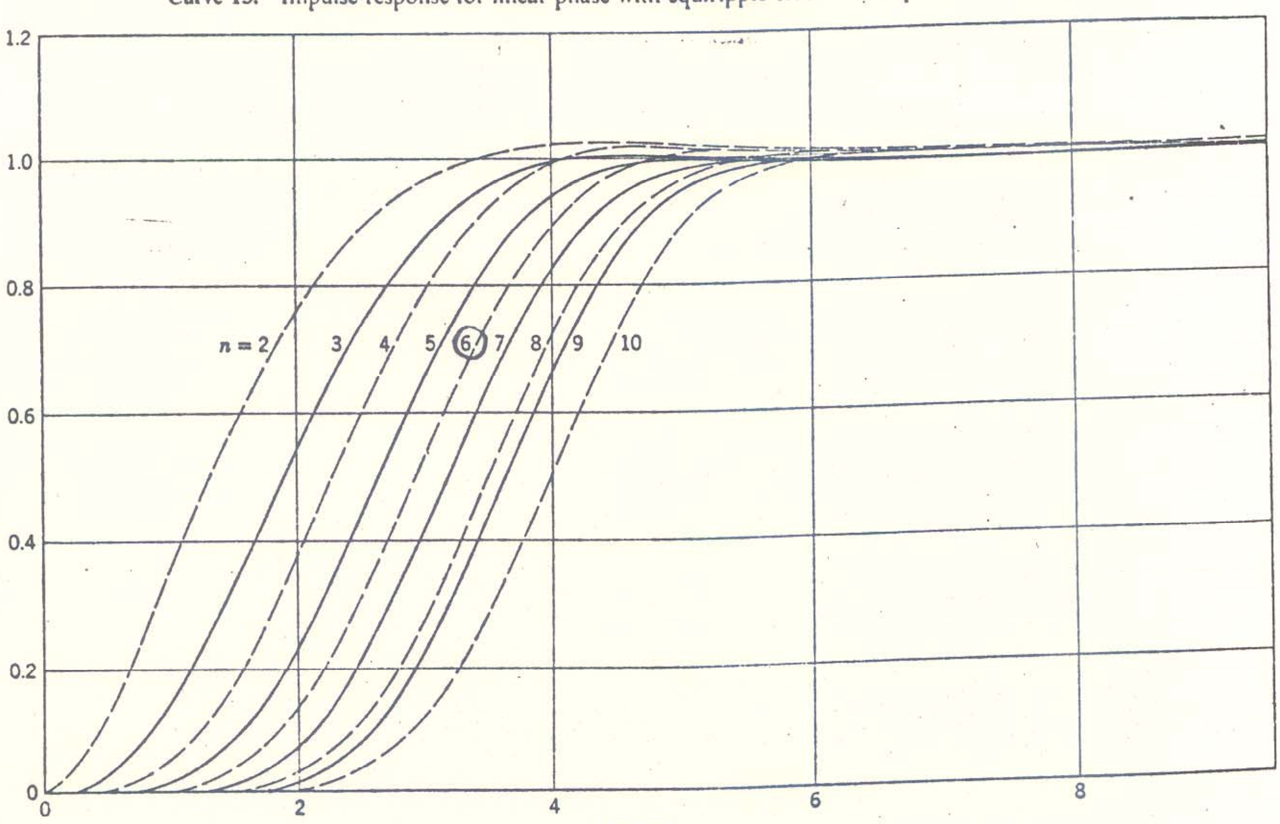


Fig 3 -

6 3.8 7.1
7 3.3

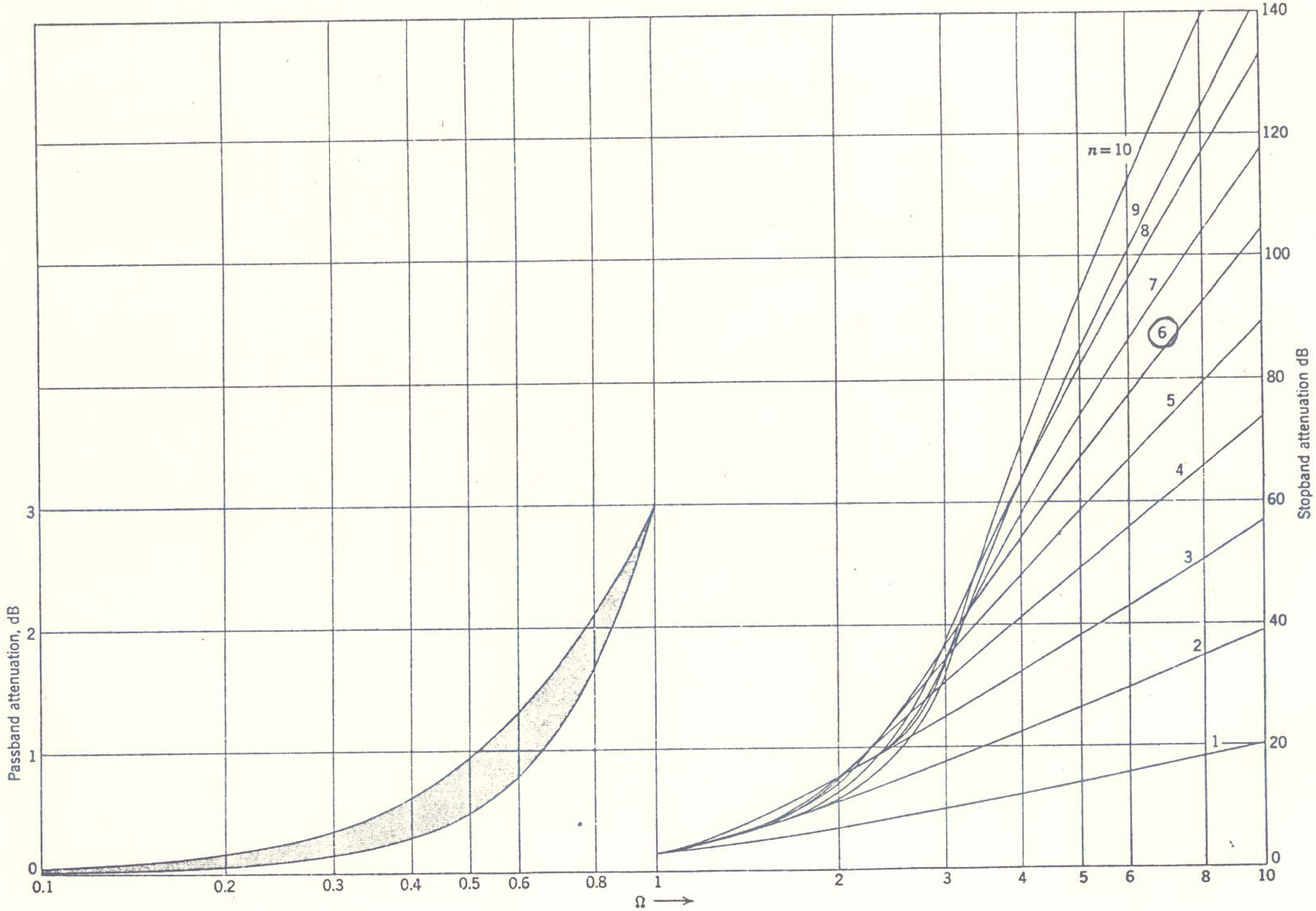


Curve 15. Impulse response for linear phase with equiripple error filters (phase error = 0.5°).

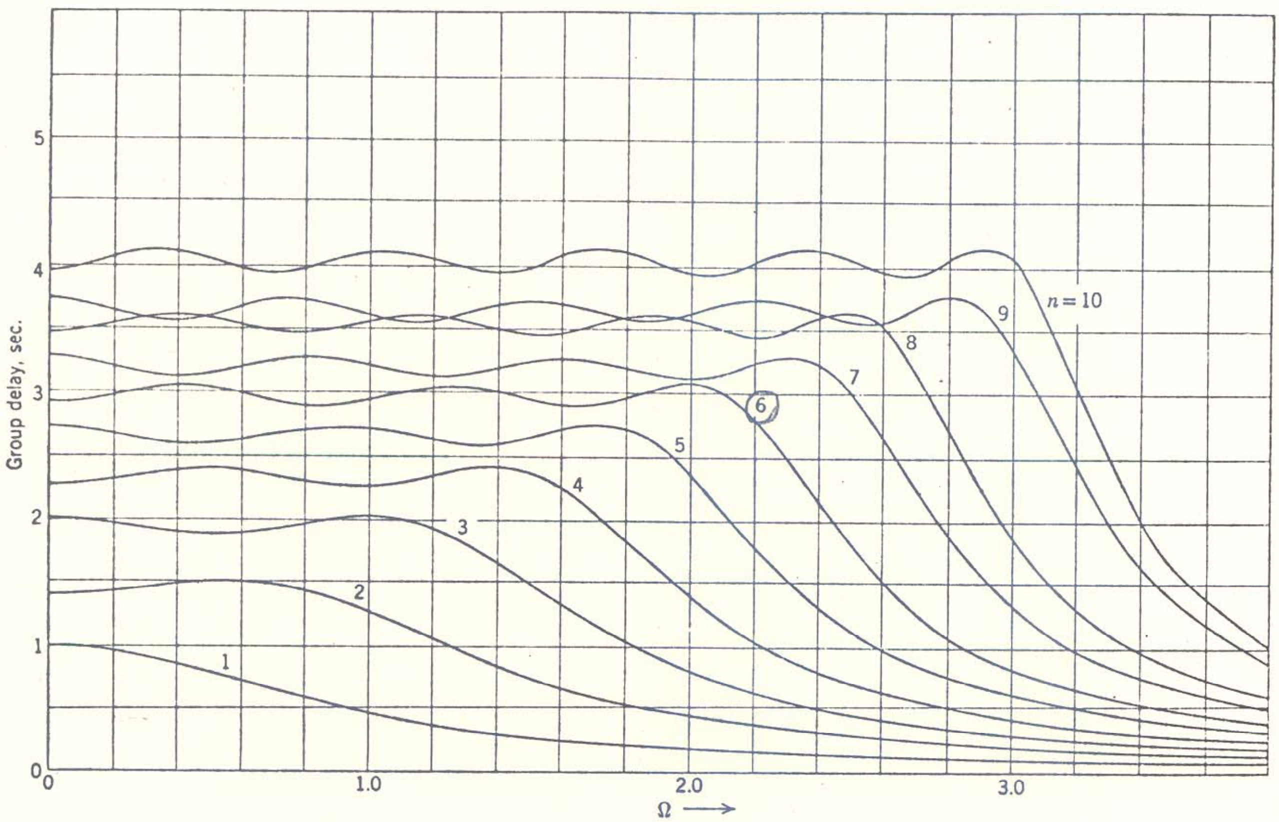


Curve 16. Step response for linear phase with equiripple error filters (phase error = 0.5°).

Fig 4



Curve 15. Attenuation characteristics for linear phase with equiripple error filter (phase error = 0.5°).

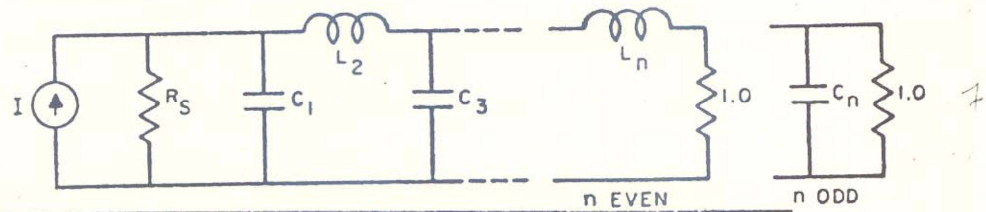


Curve 16. Group-delay characteristics for linear phase with equiripple error filter (phase error = 0.5°).

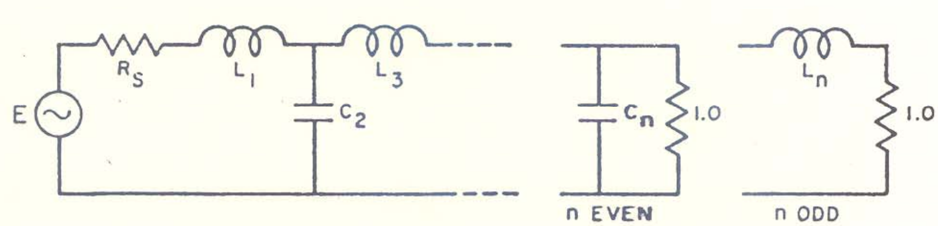
LOW PASS ELEMENT VALUES

LINEAR PHASE WITH
EQUIRIPPLE ERROR

PHASE ERROR = 0.5°



n	R_S	C_1	L_2	C_3	L_4	C_5	L_6	C_7
	1.0000	0.3658	0.6768	0.9513	1.0113	2.4446		
	0.9000	0.4027	0.6099	1.0486	0.9157	2.6062		
	0.8000	0.4485	0.5427	1.1700	0.8182	2.8114		
	0.7000	0.5069	0.4752	1.3260	0.7189	3.0787		
	0.6000	0.5843	0.4074	1.5341	0.6181	3.4387		
5	0.5000	0.6921	0.3395	1.8253	0.5160	3.9462		
	0.4000	0.8530	0.2714	2.2623	0.4130	4.7108		
	0.3000	1.1201	0.2033	2.9908	0.3094	5.9881		
	0.2000	1.6524	0.1352	4.4478	0.2057	8.5444		
	0.1000	3.2454	0.0674	8.8185	0.1024	16.2117		
	INF.	1.5327	1.0180	0.8740	0.6709	0.3182		
	1.0000	0.3313	0.5984	0.8390	0.7964	1.2734	2.0111	
	1.1111	0.2934	0.6667	0.7446	0.8985	1.1050	2.2282	
	1.2500	0.2571	0.7515	0.6542	1.0223	0.9549	2.4742	
	1.4286	0.2219	0.8600	0.5666	1.1787	0.8164	2.7718	
	1.6667	0.1876	1.0040	0.4812	1.3848	0.6859	3.1529	
n= 6	2.0000	0.1541	1.2051	0.3976	1.6709	0.5615	3.6720	
	2.5000	0.1216	1.5058	0.3155	2.0972	0.4420	4.4362	
	3.3333	0.0898	2.0058	0.2347	2.8044	0.3266	5.6935	
	5.0000	0.0589	3.0038	0.1553	4.2137	0.2146	8.1871	
	10.0000	0.0290	5.9928	0.0771	8.4320	0.1058	15.6296	
	INF.	1.4849	1.0430	0.8427	0.7651	0.5972	0.2844	
	1.0000	0.2826	0.5332	0.7142	0.6988	0.9219	0.9600	2.4404
	0.9000	0.3118	0.4802	0.7896	0.6322	1.0137	0.8718	2.5953
	0.8000	0.3481	0.4271	0.8836	0.5649	1.1287	0.7809	2.7936
	0.7000	0.3945	0.3739	1.0043	0.4967	1.2768	0.6875	3.0535
	0.6000	0.4560	0.3206	1.1650	0.4277	1.4750	0.5919	3.4051
7	0.5000	0.5416	0.2671	1.3899	0.3580	1.7531	0.4947	3.9025
	0.4000	0.6695	0.2136	1.7271	0.2874	2.1714	0.3961	4.6534
	0.3000	0.8819	0.1601	2.2890	0.2163	2.8700	0.2969	5.9091
	0.2000	1.3054	0.1066	3.4127	0.1445	4.2690	0.1974	8.4236
	0.1000	2.5731	0.0532	6.7835	0.0724	8.4691	0.0983	15.9666
	INF.	1.5079	0.9763	0.8402	0.7248	0.6741	0.5305	0.2532
n	$1/R_S$	L_1	C_2	L_3	C_4	L_5	C_6	L_7



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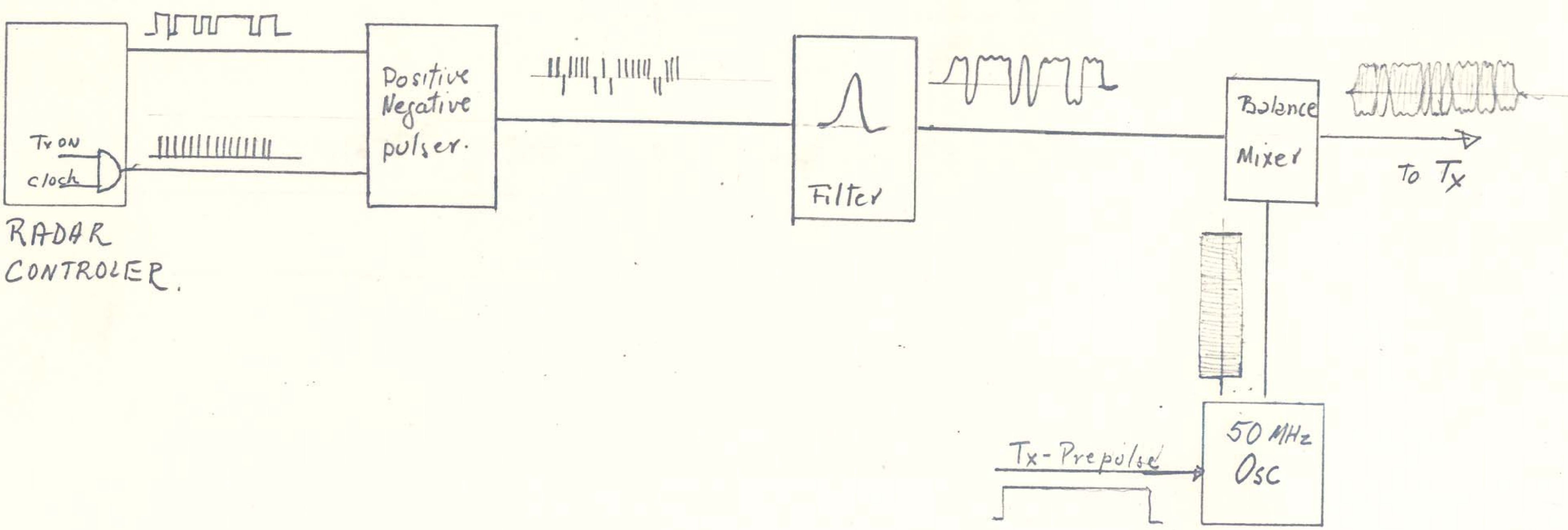

Fig 8 - Schematic diagram showing generation of Transmitter phase-coded signals matched to receiver-decoder