

Radar imaging with compressed sensing

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[1] A novel technique for radar-imaging inversions is proposed which leverages ideas from the emerging field of compressed sensing. This new method takes advantage of the transform sparsity inherent in natural images. Theoretical recovery results are promising and are borne out by simulations in which this technique outperforms Capon's method and the Maximum Entropy method. Preliminary results using data collected at the Jicamarca Radio Observatory are also presented.

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1. Introduction

[2] This paper introduces a novel technique for inverting radar-imaging data using signal processing ideas developed in the past few years known as “compressed sensing” or “compressive sampling.” Radar imaging is a technique to obtain the spatial distribution of scatterers in a direction transverse to propagation by using multiple receiving antennas. A single receiver can only produce range-time-intensity plots, which exhibit an ambiguity between spatial and temporal variation. For studying ionospheric irregularities in the equatorial electrojet and equatorial spread- F , for example, this ambiguity is not negligible.

[3] Radar imaging is a generalization of early work in two-antenna radar interferometry by *Woodman* [1971] and *Farley et al.* [1981]. *Kudeki and Sürücü* [1991] extended this work to true radar imaging by considering the use of multiple antennas. To overcome the sidelobe artifacts of the simple Fourier inversion they used, *Hysell* [1996] applied the Maximum Entropy method (MaxEnt) to radar imaging, and *Palmer et al.* [1998] proposed the use of Capon's method of imaging, all described in more detail below. A comparison of these three methods for a single- and double-Gaussian image model is given by *Yu et al.* [2000], who concluded that the direct Fourier inversion is inferior, and while MaxEnt and Capon's method are comparable, MaxEnt appears slightly superior at low signal-to-noise ratio (SNR). Another simulation study by *Chau and Woodman* [2001] reached similar conclusions. A review of radar-imaging theory is given by *Woodman* [1997].

[4] The compressed-sensing technique for radar imaging offers an alternative approach to radar-imaging analysis using familiar ideas from image compression and transform

coding. Furthermore, this technique allows a robust and intuitive way of enhancing certain desired signal features, or incorporating prior information, by choosing an appropriate sparsity basis. Compressed sensing has been found useful in the related problem of radio astronomy [*Wiaux et al.*, 2009a, 2009b; *McEwen and Wiaux*, 2011].

[5] In section 2, the radar-imaging problem is described and various methods for its solution are reviewed. The theory of compressed sensing is detailed in section 3, and the application to radar imaging is discussed. Section 4 describes a simulation study that compares this new method to current methods. Actual data from the Jicamarca Radio Observatory are shown as a proof of concept. Finally, implications and ideas for future work are described in section 5.

2. Background

[6] Radar imaging is a technique for obtaining three-dimensional images of coherent backscatter. The first dimension is range and the second and third are angles from zenith which are obtained through interferometry by combining signals from multiple spatially separated receiving antennas. In this paper only one angular dimension will be considered; the other dimension is an immediate generalization.

[7] It is a well-known result that the cross-correlation of the signals from two antennas yields a sample of the Fourier transform of the angular distribution of scattered power [*Woodman*, 1997]. This can be expressed as

$$\langle s_x, s_{x+d} \rangle \equiv g(kd) = \int_{\phi_1}^{\phi_2} f(\phi) e^{ikd\phi} d\phi \quad (1)$$

where:

[8] 1. s_x and s_{x+d} are signals obtained from antennas separated by a distance d ,

[9] 2. $\langle \cdot, \cdot \rangle$ is the cross-correlation operator,

[10] 3. k is the wave number,

[11] 4. $g(kd)$ is known as the visibility function,

[12] 5. $f(\phi)$ is the angular scattered power distribution, known as the brightness function, and

[13] 6. the integral is over a small range of angles for which the small angle approximation applies. This restriction is a result of the narrow antenna beam width.

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[14] For each range gate and for each Doppler bin, there are M instances of equation (1), where M is the number of unique antenna pairs. The radar-imaging problem is to estimate the brightness function $f(\phi)$ from these M measurements.

2.1. Direct Fourier Inversion

[15] *Kudeki and Sürücü* [1991] originally proposed a direct inversion of equation (1) using the inverse Fourier transform. However, since $g(kd)$ is only known for certain d , the problem is underdetermined. Implicit in the inverse discrete Fourier transform used by *Kudeki and Sürücü* [1991] is the assumption that the values of the unmeasured antenna separations are zero. This approach is equivalent to beam steering, sweeping the synthesized beam in software to build an estimate of $f(\phi)$. As such, the recovered image is the convolution of the true image with the beam pattern, which limits the achievable resolution. As suggested by *Kudeki and Sürücü* [1991], more advanced methods are required in order to overcome this limitation. These are discussed below.

2.2. Capon's Method

[16] Capon's method [*Capon*, 1969] was applied to radar imaging by *Palmer et al.* [1998] and can be seen as an extension of the beam steering approach implicit in direct Fourier inversion. It sweeps over ϕ to build $f(\phi)$, choosing the antenna weights at each angle in order to minimize side-lobe interference. Of all linear estimates, Capon's method produces the minimum mean-squared-error estimate of $f(\phi)$.

2.3. Maximum Entropy Method

[17] MaxEnt is best understood in the discrete domain. The discretization of equation (1) is

$$\mathbf{g} = \mathbf{H}\mathbf{f} \quad (2)$$

where $\mathbf{g}$ is a column vector comprising the M measured cross-correlations (samples of $g(kd)$), $\mathbf{f}$ is a column vector of length N representing the discretized and finite support image $f(\phi)$, and $\mathbf{H}$ is the $M \times N$ matrix operator that results from approximating the integral as a summation. The value of N sets the resolution and should be chosen so that the grid size is smaller than features of the image.

[18] In all practical cases, this problem is underdetermined, so $M < N$. Thus, there are infinite possible image vectors, $\mathbf{f}$, that agree with the data, $\mathbf{g}$. Of all possibilities, MaxEnt chooses the image with the maximum entropy or minimal information content. Proponents of this principle [e.g., *Hysell*, 1996; *Gull and Daniell*, 1978; *Jaynes*, 1988] claim that it leads to the solution that is least committal to unmeasured data and that to choose a solution with lower entropy is to imply information not contained in the data. Critics of this principle [e.g., *Cornwell and Evans*, 1985] claim that there is nothing in the radio interferometry problem resembling information; the practical success of MaxEnt arises simply from its effect as a nonnegative regularizer instead of its roots in information theory.

3. Compressed Sensing

[19] As described in section 2, the radar-imaging problem is to reconstruct a signal from a small number of samples of

its Fourier transform. The emerging theory of compressed sensing (also known as compressive sampling) offers a new perspective on problems of this type [*Romberg*, 2008; *Candès and Wakin*, 2008]. Traditionally, common knowledge held that exact recovery of an arbitrary signal was only possible if the signal was sampled at twice its bandwidth. This is the well-known Shannon-Nyquist sampling theorem. However, it has long been known that for signals that do not fully occupy the spectrum, fewer measurements are required for exact recovery. Compressed sensing formalizes these ideas by generalizing the idea of "bandwidth in Fourier space" to "sparsity of linear transform coefficients." Theoretical compressed-sensing results show that exact signal recovery is possible under much looser constraints than Shannon-Nyquist, provided that the signal is sparse when expressed in some known basis. Furthermore, certain robustness and closeness results show that this problem is well posed even when the signal is only approximately sparse and/or noisy.

3.1. Basic Theory

[20] The discrete form of our problem is given by equation (2). Despite the fact that this system is underdetermined, *Candès* [2006] shows that $\mathbf{f}$ is exactly recoverable under two conditions. First, $\mathbf{f}$ must be K sparse, meaning it has exactly K nonzero elements, where $K < N$. Second, the sensing matrix, $\mathbf{H}$, must satisfy the Restricted Isometry Property (RIP), which roughly requires that any K columns of $\mathbf{H}$ are approximately orthogonal.

[21] The naive procedure to recover $\mathbf{f}$ is direct sparsity regularization:

$$\begin{aligned} &\text{minimize: } \|\mathbf{f}\|_0 \\ &\text{subject to: } \mathbf{g} = \mathbf{H}\mathbf{f} \end{aligned} \quad (3)$$

where $\|\mathbf{f}\|_0$ is the " ℓ_0 norm" of $\mathbf{f}$, the number of nonzero elements. This procedure recovers the simplest, most compressed version of $\mathbf{f}$ that is compatible with the data. However, this problem is known to be NP-hard and therefore computationally intractable for all but the smallest problems.

[22] Remarkable results from *Candès* [2006] and *Donoho* [2006] show that the solution to problem (3) is also the solution to a more computationally attractive problem. If the RIP is satisfied, problem (3) is equivalent to an ℓ_1 norm minimization problem:

$$\begin{aligned} &\text{minimize: } \|\mathbf{f}\|_1 \\ &\text{subject to: } \mathbf{g} = \mathbf{H}\mathbf{f} \end{aligned} \quad (4)$$

This problem is often referred to as Basis Pursuit. Note that no knowledge of the number, locations, or values of nonzero elements of $\mathbf{f}$ is required. As long as the RIP is satisfied and $\mathbf{f}$ is actually sparse, ℓ_1 minimization will recover $\mathbf{f}$. These types of problems can be solved efficiently by any number of methods, such as linear programming. The fact that ℓ_1 minimization is a substitute for sparsity is one of the most practically important results of compressed sensing.

[23] This theory can be easily generalized for $\mathbf{f}$ that are not directly sparse but rather sparse when expressed in some other basis. For example, these results hold for vectors that can be described by a small number of Fourier components or wavelets. The generalization is simply to define

$$\mathbf{f} = \Psi\mathbf{s} \quad (5)$$

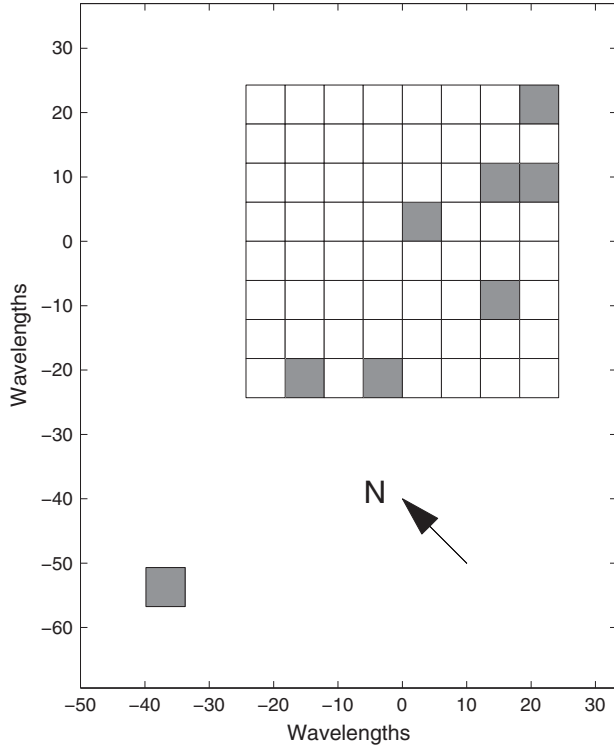


Figure 1. The antenna array at the Jicamarca Radio Observatory. In gray are the positions (measured in wavelengths) of receiving antennas in the imaging configuration.

where Ψ is an $N \times N$ orthogonal matrix that defines the sparsity basis of $\mathbf{f}$. It is chosen so that $\mathbf{s}$, the transformation of $\mathbf{f}$ into the Ψ domain, is sparse. Substituting this into (4), the problem becomes

$$\begin{aligned} &\text{minimize: } \|\mathbf{s}\|_1 \\ &\text{subject to: } \mathbf{g} = \mathbf{H}\Psi\mathbf{s} \end{aligned} \quad (6)$$

This problem is identical to problem (4). Once the solution $\mathbf{s}$ is found, $\mathbf{f}$ can be recovered using equation (5).

[24] Of course, exact recovery requires the matrix $\mathbf{H}\Psi$ to satisfy the RIP. *Candès* [2006] shows that the RIP is satisfied if

$$K \leq C \frac{1}{\mu^2} \frac{M}{\log^4 N} \quad (7)$$

for some constant C , where μ is the “mutual coherence” between the sensing matrix, $\mathbf{H}$, and the sparsity matrix, Ψ , defined by

$$\mu \equiv \sqrt{N} \max_{ij} |\langle h_i, \psi_j \rangle| \quad (8)$$

where $\langle h_i, \psi_j \rangle$ is the inner product between a row of $\mathbf{H}$ and a column of Ψ .

[25] Although the constant C is unknown and improvements to this inequality have since been proposed, the important conclusion is that the less coherent the sparsity and sensing matrices are, the fewer measurements are needed for exact signal recovery. The idea of incoherence can be understood as an uncertainty principle. Incoherence is a measure of how “spread out” the basis vectors from $\mathbf{H}$ are when transformed into the Ψ basis and vice versa. For example, the Fourier basis and the Dirac basis are maximally incoherent, since a delta function in one domain is spread out over the

entire other domain. This requirement is important because it guarantees that information from a sparse signal, $\mathbf{s}$, is spread out over all of the measurements. If the two matrices are incoherent, then with high probability, a very small number of measurements will contain much information about a sparse signal, enabling exact recovery. As a negative example, assume that $\mathbf{H}$ and Ψ are maximally coherent, i.e., the rows of $\mathbf{H}$ are columns of Ψ . In this case, the sampling is performed directly in the sparsity domain. Thus, there is no hope of an exact recovery unless $\mathbf{s}$ is fully sampled. Sampling only a few locations would likely produce all zeros. This is in opposition to the incoherent case, where the combination of a few locations is likely to contain information from the entire vector $\mathbf{f}$.

3.2. Relationship to the CLEAN Algorithm

[26] The CLEAN algorithm [*Högbom*, 1974] is a common method for processing radio astronomy data, a task closely related to the radar-imaging problem. Although this ad hoc procedure was developed more than 30 years before the discovery of compressed sensing, it can be recognized in its most basic form as an implementation of Orthogonal Matching Pursuit, an algorithm to solve Basis Pursuit problems. CLEAN attempts to find a set of point sources in an astronomical image that explains the data well. Since the problem is underdetermined, there are many such solutions. However, CLEAN implicitly finds the sparsest solution by successively introducing point sources until some convergence criterion is reached. Thus, CLEAN is implicitly solving problem (6) with Ψ as the Dirac basis, i.e., the identity matrix, which is a reasonable sparsity basis since the image comprises stars.

3.3. Statistical Estimation

[27] The compressed-sensing formulation in section 3.1 uses two impractical assumptions: the data are noiseless, and the signal is exactly sparse. In any practical scenario, the data are corrupted by noise and the chosen sparsity basis only approximately sparsifies the signal, which means that the transformed signal consists of a small number of important coefficients and a large number of negligible (but nonzero) coefficients. Due to the underdetermined nature of the problem, one might expect that with these extensions, the problem becomes ill-posed and the solution diverges wildly from the ideal as these impractical assumptions are relaxed. Surprisingly, this is not the case.

[28] If the data are noisy, the constraint in problem (6) must be relaxed from exact equality to a bound on the difference. Instead of requiring the image to exactly match the data, the value of χ^2 is bounded to within an estimate of the noise power, ϵ^2 , yielding a relaxed formulation of the problem:

$$\begin{aligned} &\text{minimize: } \|\mathbf{s}\|_1 \\ &\text{subject to: } \|\mathbf{g} - \mathbf{H}\Psi\mathbf{s}\|_2^2 \leq \epsilon^2 \end{aligned} \quad (9)$$

[29] Problems of this type are often referred to as Basis Pursuit Denoise. In this case, *Candès et al.* [2006] show that under some conditions related to the RIP, the worst case reconstruction error is bounded by a linear function of the noise level:

$$\|\mathbf{s}^* - \mathbf{s}\|_2 \leq C_1 \epsilon \quad (10)$$

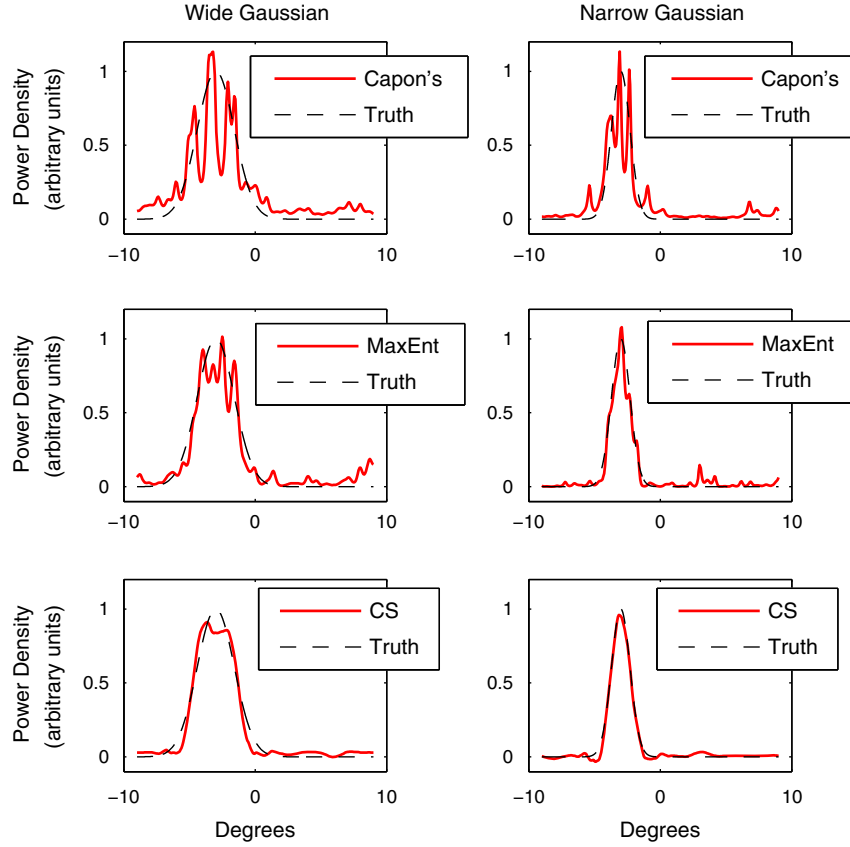


Figure 2. Results of three methods of inverting simulated radar-imaging data for two different truth images: (left) a wide Gaussian and (right) a narrow Gaussian. Statistical errors are included; the magnitude of the introduced error corresponds to a SNR of 15 dB.

where $\mathbf{s}^*$ is the reconstruction, $\mathbf{s}$ is the truth, and C_1 is a constant. *Candès et al.* [2006] find that for problems with reasonable incoherence, typical values of C_1 are around 10. Thus, despite intuition about solutions to underdetermined problems diverging in the presence of noise, sparse recovery of this form is stable and, since the solution to problem (9) is unique, well posed. The formulation in problem (9) is appropriate for uncorrelated measurement errors. In cases where the errors are correlated and have a known covariance matrix, the noise can be whitened with a linear transformation, as applied to MaxEnt radar imaging by *Hysell and Chau* [2006].

[30] As mentioned above, another practical concern relates to nonsparse signals. Although many natural signals are compressible, meaning that they can be approximated by a small number of coefficients in some basis, they are not exactly sparse. The lower order coefficients are small but not zero. In these cases, it is still possible to use compressed-sensing algorithms to reconstruct signals, but the recovery will produce a sparse approximation to the signal rather than the signal itself. In this case, the reconstruction error obeys [*Candès et al.*, 2006]

$$\|\mathbf{s}^* - \mathbf{s}\|_2 \leq C_1 \epsilon + C_2 \frac{\|\mathbf{s} - \mathbf{s}_K\|_1}{\sqrt{K}} \quad (11)$$

where $\mathbf{s}_K$ is the best K -sparse approximation to $\mathbf{s}$, that is, the vector that is created by taking only the K largest values of $\mathbf{s}$. As above, C_2 is empirically a reasonably small

constant. The reconstruction error is bounded by two terms: one related to the noise, and the other related to the inevitable error associated with approximating $\mathbf{s}$ as sparse. Thus, our problem is indeed well posed, even with signals that are not exactly sparse.

[31] Further intuition can be gained on this point by considering the related concept of image compression, specifically the JPEG-2000 compression standard. Roughly, this compression procedure performs a wavelet transform on an image and stores only the largest coefficients. This yields a sparse approximation to the image, which maintains a large percentage of the signal energy using only a small percentage of the data in the full image. With compressed sensing, the difference is that the locations of the important coefficients are not known in advance, so it would not be useful to sample the wavelets directly. Rather, sampling is performed in a different, incoherent basis to gather as much signal energy as possible. It is as if the measurements encode a compressed version of the signal, and decoding is simply a matter of choosing the sparsity basis. Clearly, exact recovery of an arbitrary signal is hopeless. However, this formulation recovers an optimally compressed version of the signal or at least as close to optimal as inequality (11) allows.

3.4. Compressed Sensing in Practice

[32] In practice, the results of compressed-sensing inversions depend heavily on the chosen sparsity basis, Ψ . This choice will depend on the properties of the signal being

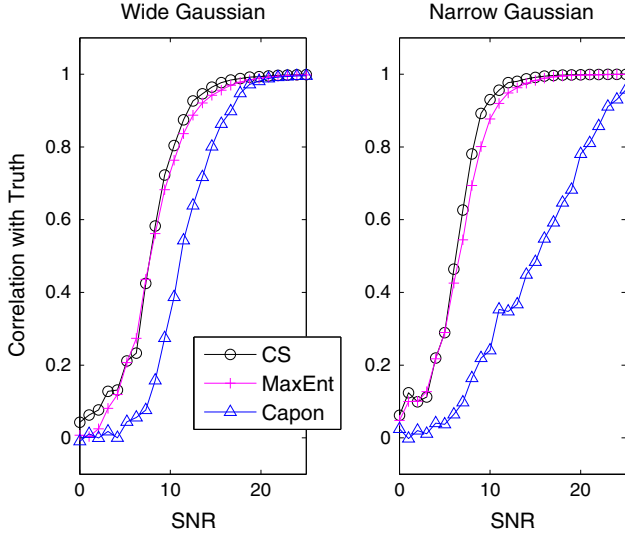


Figure 3. A comparison of the three methods of inverting simulated radar-imaging data as a function of SNR for two different truth images: (left) a wide Gaussian and (right) a narrow Gaussian. The quality metric is correlation.

imaged as well as a possible desire to enhance specific features of the image under investigation. For example, astronomers could use the Dirac basis, as their images tend to consist of point sources. Another example is to choose one of the wavelet bases, which are known for encoding step functions efficiently, a desirable property for enhancing and localizing edges in an image and one of the reasons the JPEG-2000 standard is popular.

[33] The theorems of compressed sensing are expressed in terms of probabilities, untestable conditions, and asymptotic behaviors with unknown constants. It is generally impossible to prove what level of sparsity is supported by a specific practical application. Thus it is necessary to resort to simulation. For example, *Candès et al.* [2006] find that for typical image-processing applications, as few as $3K$ incoherent measurements can reconstruct images with better accuracy than knowing all N coefficients and using the largest K . For the radar-imaging application in the present paper, the efficacy of the proposed method must be shown by simulation.

4. Results

[34] A simple radar-imaging experiment was simulated in order to compare the compressed-sensing method to other popular methods. For simplicity and to match with the intended application of imaging equatorial ionospheric irregularities, only one dimension is simulated, parallel to magnetic longitude. Stacking the retrieved 1-D angular distributions from each range gate creates a 2-D image of the equatorial ionosphere. It is assumed that irregularities map along magnetic field lines [Farley, 1960]. Antenna locations are used from the imaging configuration of the Jicamarca Radio Observatory located near Lima, Peru. This setup is shown in Figure 1. The magnetic longitudes of the antennas are used to determine the relevant baseline lengths.

[35] Equation (2) is used as a forward model, and an error term is added to simulate the statistical uncertainty

associated with estimating the cross-correlation with finite integration times. A high-resolution grid ($N = 2^{14}$) is used to simulate measurements, and a coarser grid ($N = 2^8$) is used for the MaxEnt and compressed-sensing (CS) reconstructions. Capon's method does not rely upon a discretization. As long as the reconstruction grid size is chosen large enough that features in the image are larger than the grid spacing, the inversion is not sensitive to it. In this study, Gaussian white noise provides the error term. The use of a more detailed forward model such as the one used in *Yu et al.* [2000] with a more detailed error model as in *Hysell and Chau* [2006] is left for a future simulation study.

[36] Following *Yu et al.* [2000], a Gaussian is used as the true image $f(\phi)$. Two cases are considered: a wide Gaussian (width 2°) and a narrow Gaussian (width 1°). Both are centered at -3° . The data are inverted with Capon's Method, the Maximum Entropy Method (MaxEnt), and the new compressed-sensing (CS) method. For both the MaxEnt and CS inversion, the value of χ^2 is constrained to be less than its expectation, in accordance with *Hysell* [1996]. Capon's Method contains no concept of such error and instead adheres exactly to the data. The CS solution is found by solving the Basis Pursuit Denoise problem, (9), for which the technique developed in *van den Berg and Friedlander* [2009] is used; a MATLAB function is available online [van den Berg and Friedlander, 2007]. For the sparsity basis, Ψ , the Daubechies-4 wavelets are used. A wavelet basis was chosen due to its connection with the JPEG-2000 compression standard, and in particular, the Daubechies-4 wavelets were chosen because they were found useful in image-processing applications [Duarte et al., 2008].

[37] Typical results using a signal-to-noise ratio of 15 dB are shown in Figure 2 for both the wide and narrow Gaussian cases. Since Capon's method does not produce meaningful absolute values, the Capon image has been scaled to match the true image in a mean-squared-error

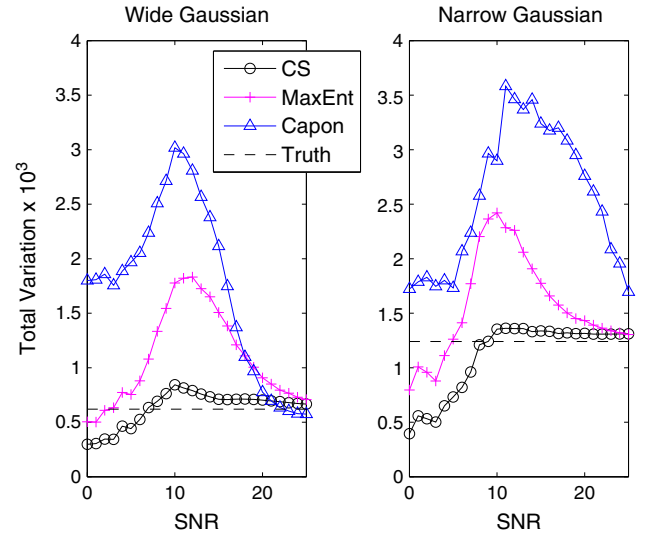


Figure 4. A comparison of the three methods of inverting simulated radar-imaging data as a function of SNR for two different truth images: (left) a wide Gaussian and (right) a narrow Gaussian. The quality metric is total variation. A smaller total variation indicates a smoother image. The total variation of the true image is also shown for reference.

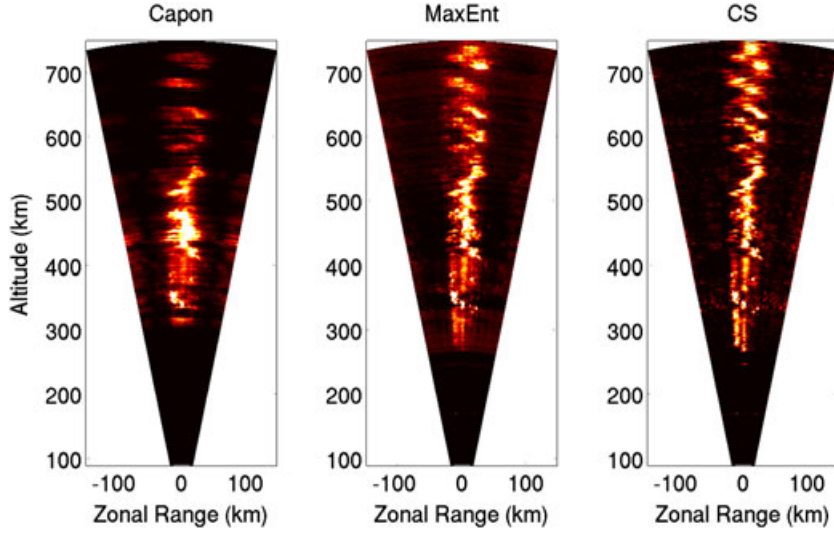


Figure 5. A comparison of the three inversion techniques using data acquired at the Jicamarca Radio Observatory during equatorial spread- F conditions. The color scale is arbitrary.

sense. The MaxEnt and CS inversions have not been modified. Qualitatively, it appears that the compressed-sensing inversion produces fewer reconstruction artifacts than the other methods, for both scale sizes. However, the quality of the CS reconstruction appears to degrade for large, slowly varying sources, as these sources are not as compressible in the wavelet domain. The running time of the CS algorithm is roughly equal to MaxEnt, but about 20 times longer than Capon's method.

[38] Quantifying image quality is a hard problem. The typical mean-squared-error metric assumes unbiased absolute values and offsets, which is not a necessary restriction for the radar-imaging problem. It is only required that the estimate portray the same morphological features as the truth. For these reasons, the normalized correlation is used. Nevertheless, qualitatively similar results are obtained with a mean-squared-error metric. The correlation is defined as

$$\frac{\mathbf{f}_{\text{est}}^T \mathbf{f}}{\|\mathbf{f}_{\text{est}}\|_2 \|\mathbf{f}\|_2} \quad (12)$$

where $\mathbf{f}_{\text{est}}$ and $\mathbf{f}$ are the estimated and true images, respectively, with means subtracted.

[39] To investigate the performance of CS, many simulations are run with values of SNR from 0 to 25 dB. For each value of SNR, the simulation described above is repeated with 200 different error realizations in order to mitigate statistical fluctuations. The average correlation is calculated for each SNR and shown in Figure 3. For both scale sizes, Capon's method is inferior for all SNR. The compressed-sensing method outperforms MaxEnt except for SNR between 5 and 7 dB, where MaxEnt has a slight advantage for both scale sizes. CS performs better for the narrow Gaussian than for the wide Gaussian, presumably because the narrow image has a sparser representation in the wavelet domain.

[40] Although the correlation metric successively measures large-scale morphology, it does not provide information on small-scale artifacts. In Figure 2, it is clear that

Capon's method and MaxEnt return images with spurious oscillations that could be mistaken for signal features, while the CS image exhibits fewer such artifacts. In order to quantify these small-scale features, the total variation (TV) of the image is calculated:

$$TV = \sum_{i=0}^{N-2} |f_{i+1} - f_i| \quad (13)$$

[41] A smoother image with fewer small-scale features has a smaller TV . For these particular truth images, which are smooth, a high TV indicates that the reconstruction contains spurious oscillations. The average TV is calculated for each SNR, and the results are shown in Figure 4. For the wide Gaussian case, CS provides a smoother image than MaxEnt or Capon's method for SNR below 21 dB. As SNR increases above 21 dB, both MaxEnt and CS converge to the TV of the true image. Capon's method does not converge to the true image and instead converges to an image with a smaller TV than the true image. For the narrow Gaussian case, CS provides the smoothest image for all SNR except 25 dB, for which MaxEnt produces images with slightly lower TV .

[42] The result of using these three techniques on actual data is shown in Figure 5. The data were collected using the 50 MHz radar at the Jicamarca Radio Observatory at 22:18:30 LT on day 277 of 2011, during equatorial spread- F conditions. An interval of 12 s is used to estimate the signal correlations. The sensing matrix, $\mathbf{H}$, and sparsity basis, Ψ , are the same as in the simulations above. An inversion is performed for every range gate and the results stacked to form the image.

[43] Capon's method suffers from blurring, possibly due to the width of the main lobe of the antenna beam. The MaxEnt image is an improvement over Capon's method but exhibits a nonzero background level in some range gates. The CS inversion appears to give the best resolution and image clarity of the three methods. However, the true image is unknown, so it is impossible to accurately assess image quality.

5. Conclusion

[44] This work introduces a new technique for processing radar-imaging data based on the emerging field of compressed sensing. Most practical radar-imaging problems are underdetermined, so regularization is necessary. MaxEnt regularizes with an entropy functional. Capon's method is not classical regularization, but it regularizes the problem by restricting it to linear processing schemes. The compressed-sensing method leverages the inherent sparsity of natural images and regularizes by choosing the sparsest, most compressed image that matches the data.

[45] The best choice of the sparsity basis remains an open question. This choice represents a way of robustly incorporating prior information or preference for certain image features, a tool unavailable in MaxEnt or Capon's method. The simulations in this work suggest that even a simple wavelet basis can outperform existing methods. A recent trend in compressed-sensing research is the use of "overcomplete" dictionaries: a sparsity matrix, Ψ , with more columns than rows, which leads to more sparsifying power at the expense of more computation. Using overcomplete dictionaries for radar imaging deserves to be investigated.

[46] Another future task is to robustly introduce a nonnegativity constraint, since scattered power cannot be negative. Both Capon's Method and MaxEnt reconstruct strictly nonnegative images. While there exist some results for nonnegative compressed sensing [e.g., O'Grady and Rickard, 2008], they have not yet been generalized to an arbitrary sparsity basis. Future work should also include simulations using more complex and realistic signals, including signals with structure in the frequency domain, and should incorporate the antenna radiation pattern in the inversion.

[47] Of further interest is whether compressed-sensing theory can aid in not only the analysis but also the design of imaging experiments. Optimal recovery can likely be achieved by choosing an antenna layout that is most incoherent with the sparsity basis. Finally, the processing time of the compressed-sensing inversions can be improved by developing more specialized methods and by using fast transforms instead of matrix arithmetic.

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