DETERMINATION OF MAXIMUM AGE FROM THE PROBABILITY DISTRIBUTION FUNCTION OF A GROWTH PARAMETER

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The present notes are motivated by the problem of trying to determine the maximum age of trees which we have under study in the northern part of Perú. The final objective of the project is to decipher the occurrence of El Niños in the past, which should have been registered in its growth rings.

The Carbon-14 technique presents difficulties in the determination of the age of samples taken in the last 400 years. The calibration curve makes an inflection which makes the determination of age, quite uncertain within this period. Any additional techniques, even if they provide gross estimates, are useful to reduce the uncertainties in the Carbon-14 ages.

The notes are further motivated by our observations that certain species show very dramatic features in their distribution of sizes. For instance, we have noticed that it is very difficult to find young Hualtacos, they must exist, and yet we have not seen one. In fact, we were made aware of this strange behavior by a native of the area which claimed he had never seen one. On the other extreme of the growth cycle, it is difficult to find dead sapotes in the desert. Both observations lead us intuitively to guess that this is a manifestation of the longevity of these species. In the present notes we proof that indeed the distribution of occurrence of any growth parameter, like trunk size, for instance, carries information about the maximum age of the specie, and that a small probability to find a given age groups is indicative of longevity.

The problem can be stated as follows: given a distribution of sizes and a assumed shape of distribution of ages, but with the temporal length scale unknown, can one determine this unknown. We will show that, in fact, such unknown is uniquely determined and the functional relationship between the size and age, including the maximum age can be evaluated.

We will assume that the populations of tree species is a time stationary random process in the time scales comparable to a significant fraction of its life time. It could vary randomly from year to year, or even a few years at a time. We will also assume that in the average there is a given but unknown relationship between growth and age. To make things easy, we will also assume it to be a deterministic one, that is, for a given age a given size. This is a good approximation if the dispersion of sizes is a small fraction of the mean. We

could alternatively, assume the growth parameter size to be a random process in time, but this extra complication, we expect, would change our conclusions in a quantitative way but not in a qualitative way. We will leave such sofistications for future work.

Let the age of a tree, ξ, be a random variable with probability distribution $F_\xi(x)$ and probability density function $P_\xi(x)$. That is

$$F_\xi(x) = \text{Probability of } 0<\xi<x \tag{1}$$

$$P_\xi(x)\Delta x = d/dx\ F_\xi(x).\Delta x \text{ Probability } x<\xi<x+\Delta x \tag{2}$$

The size of the tree (or any other parameter which monotically increases with age), η, is another random variable with $F_\eta(y)$ and $P_\eta(y)$ for probability distribution and density function respectively. We will assume, as stated, that $\xi = \xi(\eta)$ or $\eta = \eta(\xi)$, which also implies a function relation between x and y, i.e. $x = x(y)$ and $y = y(x)$, where x and y correspond to a given age and size, respectively.

We have that

$$F_\eta(y) = F_\xi(x) \quad \text{for} \quad x = x(y), \tag{3}$$

or

$$P_\eta(y) = d/dy.F_\xi(x),$$

or

$$P_\eta(y) = d/dx.F_\xi(x).dx/dy$$

or, finally that

$$P_\eta(y) = P_\xi(x).dx/dy \tag{4}$$

Equation (4) is a differential equation where $P_\eta(y)$ is known, evaluated empirically by means of field measurements, with a given solution, if $P_\xi(x)$ is assumed known. The solution is given as a function relating x with y. The constant of integration is evaluated from $y(0) = 0$. Problems of interest are those where $P_\xi(x)$ is known in shape, but in terms of an unknown set of parameters.

Note that the solution of (4) is no other than (3), which is obtained by integration. If both distributions were analytical functions of x and y, it would give us an equation which relates x and y.

In practice, in the tree population problem, one has an empirical knowledge of $P_\eta(y)$ or F_η, as a tabulation of values; but one ignores x(y) and $P_\xi(x)$ [or $F_\xi(x)$], in which case both equations (3) and (4) are of little value. On the other hand, theoretical considerations can be used to obtain reasonable function shapes for y(x) or for $P_\xi(x)$, but with unknown coefficients. For instance, one could estimate that the growth is a quadratic function of age, i.e. $y = ax^2$; or that all trees have the same growth and life cycle but with a random stationary birth date. If in the latter case one assumes, in addition, that the death rate is zero, except at a maximum age, T_{max}, $P_\xi(x)$ is then a constant given by

$$P_\xi(x) = 1 / T_{max} \qquad (5)$$

With either assumption, one can find a solution. Either $F_\xi(x)$, if y(x) is assumed, or x(y), if $F_\xi(x)$ [or $P_\xi(x)$] is assumed. In both cases, one needs one known point to solve for the parameter constant (a or T_{max} in above example).

In the more general case, N parameter constants need to be evaluated. In this case, N calibrated points will provide the N required equations for their evaluations.

We can illustrate the procedure assuming equation (5) to be valid. By integration we find that $F_\xi(x) = x/T_{max}$. Using (3) we get

$$x = T_{max}\, F_\eta(y) \qquad (6)$$

$F_\eta(y)$ is obtained empirically as a tabulation of values, for a selected number of values of y, which in turn will give us a tabulated function $x_i(y_i)$, within a multiplicative factor, T_{max}. We need to know only one point on the function $x_i(y_i)$ to obtain T_{max}. This point could be, for instance, the size of a tree of three years of age, and would give us trough (6), the maximum age of the tree. In the particular problem that motivated this note, this parameter is the one we were mainly interested in. We could determine, then, by observing only a few years, maximum ages of the order of several hundred years.

Of course, the validity of our conclusions would depend on the validity of our assumptions. On the other hand, it is expected that reasonable assumptions would ,at least, give us right order of magnitude estimates of the functions parameters, like T_{max}, for instance. Furthermore, one can introduce more than one parameter in the definition of function shapes and get closer to the real functions. For instance, one can assume a cubic growth rate ($y = ax+bx^2 +cx^3$) or a variance in the size for a given age or a mortality rate as a function of age. A larger numbers of parameters involves a larger number of points in a know region of $P_\xi(x)$ and $P_T(y)$. Once they are determined, one should be able to get better estimates of our goal which would be the corresponding parameter to T_{max}.

Continuing with our illustration, let us invent, in lack of real data at the moment, a density distribution function for Hualtacos, to look like figure (2). Although the data is the product of our imagination, it conforms with the real distribution in the sense that we have never seen young trees with diameters less than 10 cms., and that we have not seen them with diameters larger than 50 cm. It also shows an even distribution with sizes between 20 and 35 cms. We have constructed the data so that there is less than 1% for trees less than 10 cms., since we have not seen more than 100 trees, it explains the fact we have not seen any that small. Let us, also, perform a (reasonable gedanken) calibration and "find" that it takes 5 years to grow the first 10 cm. Since $F_\eta(10) = 0.01$ and $x(10) = 5$ years, from (6) we get that $T_{max} = 500$ years.

Figure 3 represents $F_\eta(y)$, the integral of P (y) shown in Figure 2. It is also the function x(y) when multiplied by T_{max} if we assume a constant $P_\xi(x)$ density distribution (solid line of figure 1). Note that Figure 3 can also be read as $y = y(x)$. Although in this case, it shows an unreasonable increase in size shortly before reaching maximum age. This a consequence of incompatibility between a constant $P_\xi(x)$ with all trees reaching T_{max} and the decay in population at larger diameters. The latter is compatible with a $P_\xi(x)$ as shown in the dotted lines of Figure 1. In any case, the conclusions derived in above example would still be the same: we can associate a T_{max} characteristic time to $P_\xi(x)$ which would be of the order of 500 years for a similar calibration point. In both cases, y(x) will show a rapid increase in early years and a slow down after "maturity" ($y \geq 10$ cm.).

Earlier in the introduction we mentioned that we had noticed that we seldom see dead sapotes and claimed that intuitively we interpret this as an indication of longetivity. To show this as an application of (3), it is better to replace the random variable η by two states A and D, alive and dead. The random variable ξ still representing the age of a tree selected at random, including the time where the tree is still observable, even if dead. We can still write

$$F_\eta(S_i) = F_\xi(x_i) \qquad \text{for } x_i = x_i(S_i)$$

$$\text{and } S_i = A \text{ and } (A.or.D)$$

where $x_i(S_i)$ is the **maximum** age of state S_i and $F_\xi(x_i)$ = Probability $[\xi < x_i(S_i)]$.
If we take that the frequency of occurrence of dead species is 1%, i.e. $P_\eta(D) = 0.01$, and the same assumption $P_\xi(x)$ = constant = $1 / T_{max}$ (all trees go through the same life cycle), or $F_\xi(x) = x / T_{max}$ we can write

$$x_\xi / T_{max} = F_\eta(S_i)$$

or

$$x_i(A) / T_{max} = F_\eta(A) = 0.99$$

$$x_i(D) / T_{max} = F_\eta(A.or.D) = 1$$

and consequently the time difference $x_i(D) - x_i(A)$ where the tree is in decomposition but still alive.

$$T_{decomp} = x_i(D) - x_i(A) = 0.01\ T_{max}$$

If we assume that the time of decomposition is of the order of 3 to 5 years (a branch cut and left in the desert, is still observable but being eaten by insects after 3 years), then we would get again a T_{max} of 300 to 500 years.

Figure 5 shows schematically above definitions. A similar plot can be made with a more realistic age probability distribution function. The conclusion would still be the same: time spans are (approximately) in proportion to the populations within that time.

CONCLUSIONS

Observations of the population density of a tree specie at different degrees of growth carry important information about the time scale of the process. The technique requires certain assumptions, but even rough assumptions give already estimates of the right order of magnitude, sufficient to reduce the range of age values obtained using C-14 techniques in the difficult period which includes the last few hundred years. Also, very rough estimates of the population of young and dead species of sapotes and Hualtacos in the Sechura desert, are giving us a few century estimates for the maximum ages of these species. Needless to say future observations should be made in a systematic way, choosing a uniform environment away from any possible man made interference, to replace our imaginary data.

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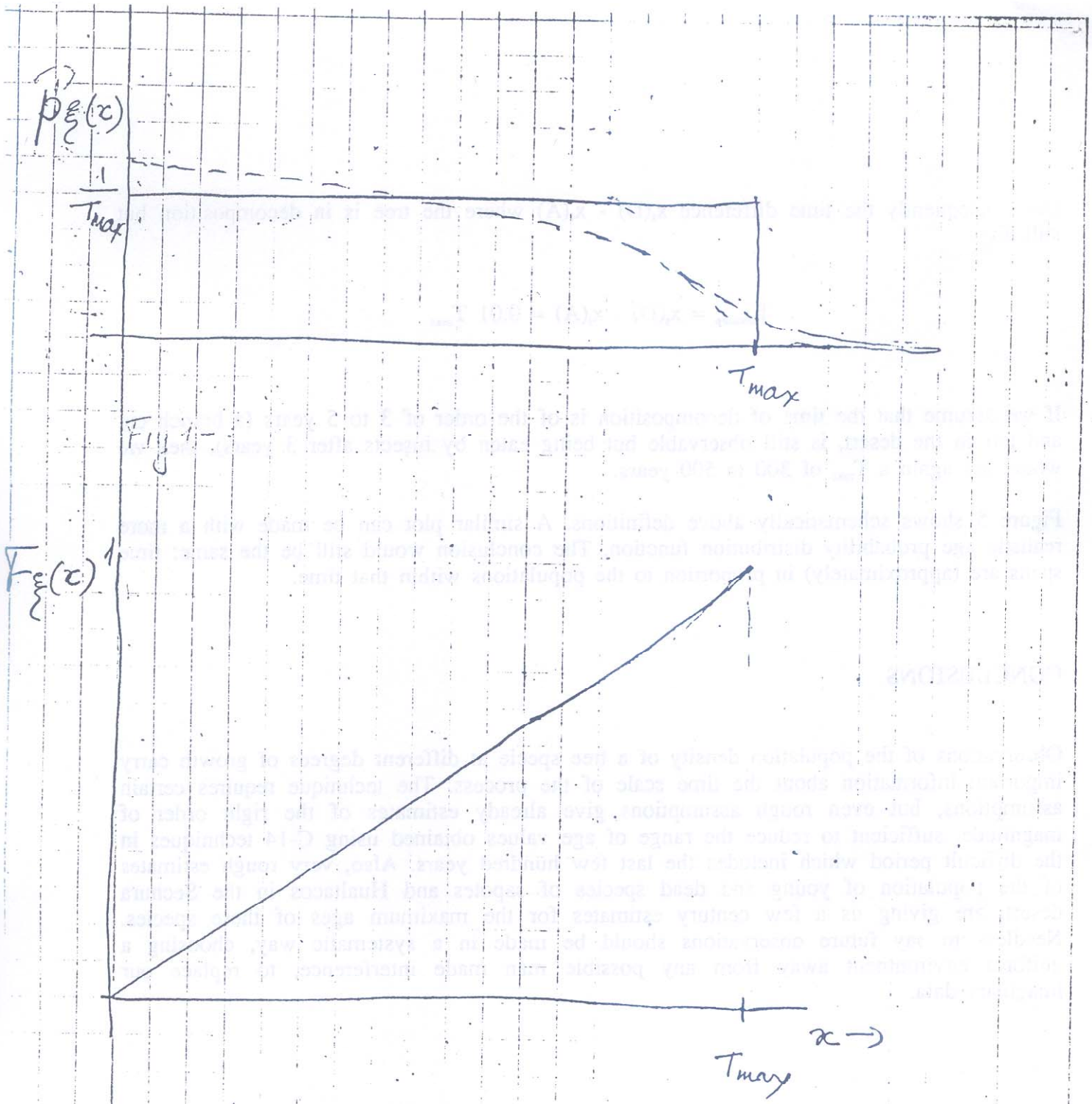


Fig 1

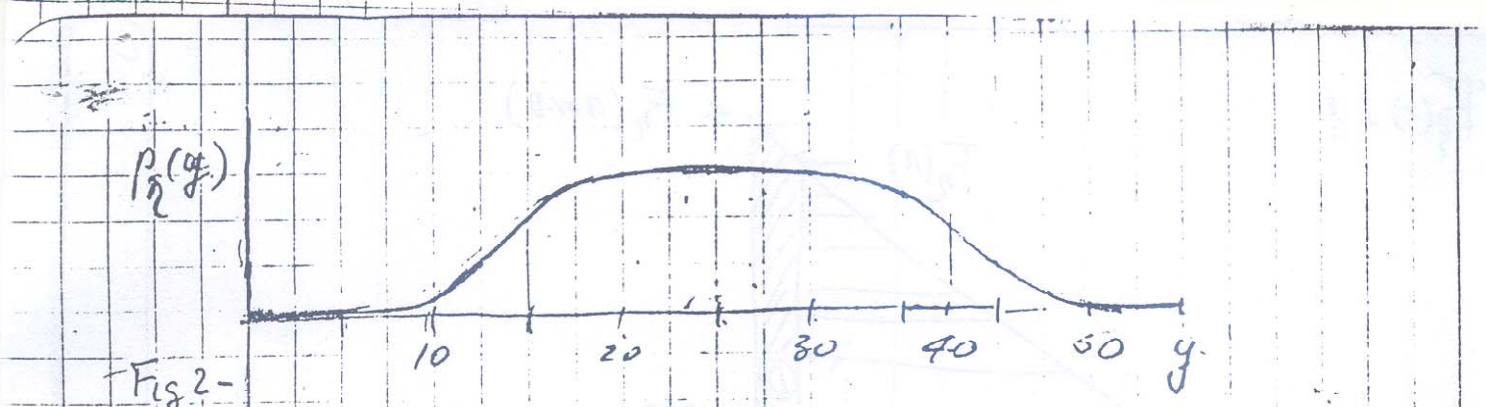


Fig 2-

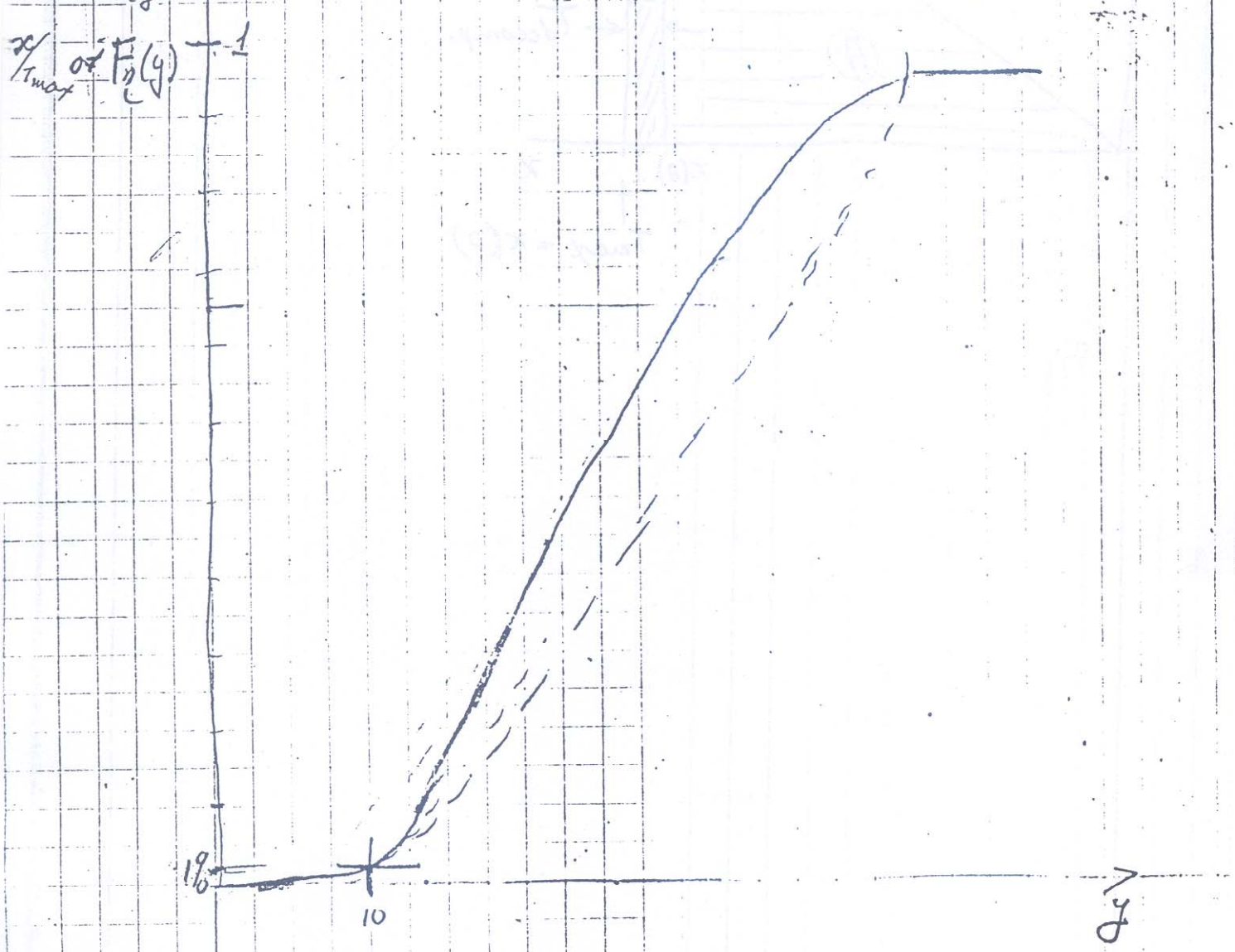
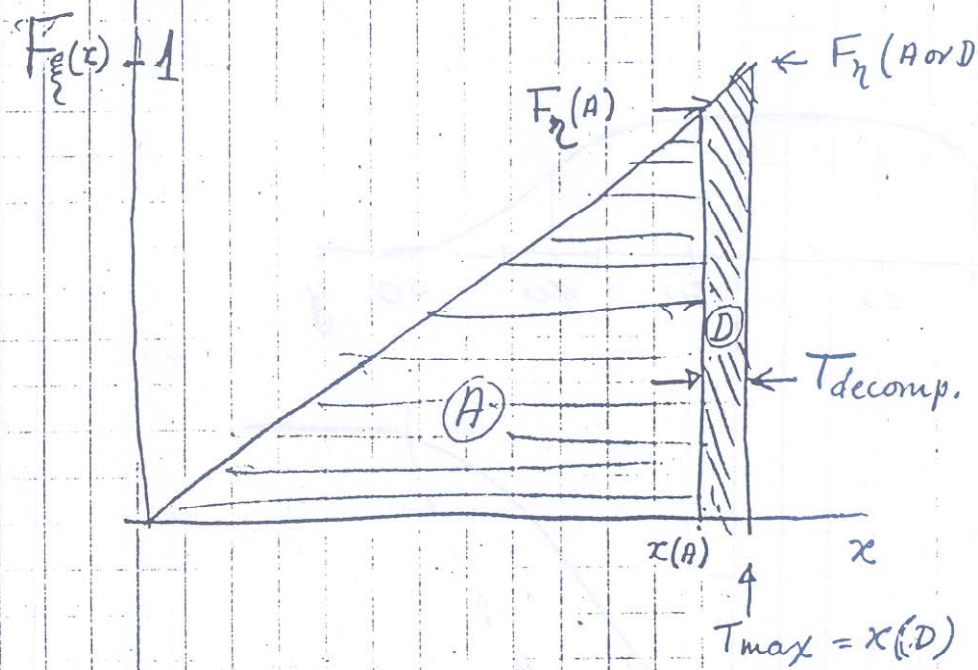


Fig 3-


$F_{\xi}(x) = 1$
$F_{\eta}(A)$
$\leftarrow F_{\eta}(A \text{ or } D)$
D
A
$\leftarrow T_{decomp.}$
$x(A)$
x
$T_{max} = x(D)$

Fig