

Demodulation of complex baseband radar signals for the analysis of multiple narrow spectral lines

Enrico Ragaini

Dipartimento di Elettrotecnica, Politecnico di Milano, Milan, Italy

Ronald F. Woodman

Radio Observatorio de Jicamarca, Lima, Peru

Abstract. This paper proposes a technique for processing signals in which the relevant spectral feature consists of one or more narrow lines which have a nonzero Doppler shift with respect to the reference frequency. The proposed technique consists of applying a second demodulation process to the demodulated signal obtained as the receiver output. The purpose of such a demodulation is to remove the frequency shift between the observed spectral line and the center of the observed spectral band. This way, the selected spectral line is brought to baseband, allowing the study of its envelope as a slowly varying process. One efficient way to perform the second demodulation step is by a digital version of a quadrature demodulator. This idea is translated into an efficient algorithm, employing real multiplications and accumulators. An advantage of such an algorithm is that in order to observe two spectral lines in symmetrical positions with respect to the center frequency, common multiplication and accumulators can be used. A possible simplification of the algorithm is discussed in which sinusoids used for demodulation are replaced by square waves, so that multiplication reduces to a synchronous sign reversal. Possible distortions due to this simplified algorithm are mentioned. Possible implementations of double demodulation in a digital signal processing system are discussed in the final part of the paper.

1. Introduction

Doppler radar experiments are based on spectral analysis. In fact, information is obtained from data by evaluation of their frequency content. For example, in a wind profiling experiment, physical quantities are obtained by evaluation of the Doppler shift and width of the frequency spectrum of clear-air echoes. In a generic Doppler radar experiment the measurement system produces a set of frequency spectra of the received signal. A narrow spectral band centered on the frequency of the transmitted signal is observed, so that frequency offsets (Doppler shifts) can be appreciated. Typical observed bandwidths range from some hertz to some hundreds of hertz. In some experiments the relevant part of the measured spectrum consists of one or more narrow spectral lines, which are removed from zero offset frequency. Our main concern in this paper is about lines with a frequency offset much larger than their spectral width. We will refer to

such spectral features as “shifted narrow lines.” We can consider such spectral features as “doubly modulated” baseband signals, in which the first frequency shift is due to the radar carrier frequency and the second to some large Doppler shift imposed by some peculiar condition of the target. Receivers remove the former frequency shift by means of a reference (carrier frequency) signal in the usual demodulation process, but the latter is still present at the receiver outputs. In these cases the physical measurement is based on the accurate evaluation of the shape and position (offset) of such lines. Line widths of fractions of a hertz are common: Typical examples are sea echoes in HF or VHF measurements and acoustic lines in radio-acoustic sounding systems. Thus a high-frequency resolution is required in spectral evaluation in order to obtain significant results.

With usual fast Fourier transform (FFT)-based spectral estimation algorithms, analyzing a shifted narrow spectral line in detail is often impossible, because the same frequency resolution has to be applied to the whole observed band, so that a large number of spectral points have to be used. The

Copyright 1997 by the American Geophysical Union.

Paper number 96RS03580.
0048-6604/97/96RS-03580\$11.00

problem arises when the spectral features are both shifted and narrow, because FFT-like algorithms process a spectral band centered on zero offset, while high-frequency resolution is needed to observe accurately the shape of narrow lines. This can easily become an overwhelming requirement for the signal processing system, especially when many range gates have to be processed in parallel (i.e., many spectra have to be calculated at once).

In this paper, we propose an easily implementable processing technique which would allow observation of narrow spectral bands without requiring extensive calculations. The main idea is to apply a second demodulation to the receiver output signals in order to bring the spectral lines of interest to baseband. The main difference between such a process and the usual quadrature demodulation is that a complex signal is demodulated instead of a real one.

2. Quadrature Demodulation and Coherent Integration

In this section, quadrature demodulation and coherent integration are briefly outlined. For a complete discussion, we refer the reader to *Rastogi* [1989] or *Keeler and Passarelli* [1990, and references therein]. The simplification of the demodulation and coherent integration process can be

The second term, a narrowband signal at frequencies around 2Ω , is filtered out, and $s(t)$ results. Quadrature demodulation is a common technique, used in many Doppler radar processing systems.

A quadrature demodulator actually works by mixing (multiplying) $r(t)$ by two sinusoids at the frequency of the local oscillator and 90° out of phase, and low-pass filtering the resulting signals, which are then called the "in-phase" and "in-quadrature" components of the signal and correspond to the real and imaginary parts of $s(t)$. Two real multiplications are thus needed. Multiplications and filtering are implemented by analog circuits. The two output signals are then sampled and used as a complex time series for spectral analysis. The sampling rate used for the demodulated signals is often much faster than the bandwidth of interest. In these cases the first processing step is a digital low-pass filtering. This can be implemented by means of time domain averaging: Blocks of successive samples are summed together, and the results are used as a new time series which is then frequency analyzed. As the real and imaginary parts of the samples are accumulated separately, phase coherence is maintained. Time averaging is also known as coherent integration, and it is currently used in many Doppler processing systems.

Demodulation and coherent integration can be

To simplify the discussion, we consider observation of only one height (a single range gate), supposing that the same processing algorithm is used for all observed heights. The signal received by the antenna is a real, narrowband (or quasi-sinusoidal) process. Its frequency spectrum is confined to a narrow band centered on a carrier frequency, namely, the frequency of the transmitted signal. This is a consequence of the physical processes involved, because the received signal can show a range of Doppler shifts, positive or negative, with respect to the transmitted frequency. Using complex notation, the received signal $r(t)$ can be written as

$$r(t) = s(t)e^{j\Omega t} + \bar{s}(t)e^{-j\Omega t} \quad (1)$$

where Ω is the frequency of the transmitted signal. We can say that $r(t)$ is an amplitude-modulated signal and Ω is the carrier frequency. Here $s(t)$ is the baseband signal. The demodulation process consists of obtaining $s(t)$ from $r(t)$. If a quadrature demodulator is used to recover $s(t)$, $r(t)$ is multiplied by $e^{-j\Omega t}$ and then lowpass filtered. Multiplication yields

$$r(t)e^{-j\Omega t} = s(t) + \bar{s}(t)e^{-j2\Omega t} \quad (2)$$

Demodulation and coherent integration can be considered together as a filtering process: The signal is first shifted down in frequency, so that the relevant frequency band lies around zero frequency. This spectral band is then selected by means of a low-pass filter.

3. Double Demodulation Concept

If the spectrum of the signal includes narrow features which are themselves shifted from the center frequency by an amount much larger than their width, a situation similar to the previous one occurs at the output of the quadrature demodulator, with the difference that $s(t)$ is now a complex shifted narrowband signal, whereas $r(t)$ was real. Let ω be the displacement of the line with respect to the center (reference) frequency. We will suppose ω to be much larger than the spectral width of the spectral feature we are interested in but much smaller than the sampling frequency. Thus the signal after demodulation can in turn be expressed as

$$s(t) = x(t)e^{j\omega t} \quad (3)$$

If ω is known (actually, it is approximately known in many cases), we can apply a second demodulation process to $s(t)$ and recover $x(t)$, although this time we are demodulating a complex signal. This can be done by a quadrature demodulation (i.e., multiplication by a complex exponential) followed by a coherent integration, which works as a low-pass filter. If signals are sampled and digitized at receiver output, i.e., after demodulation, both steps would be performed by digital circuits, as $s(t)$ is available to the signal processing system in the form of a complex time series. The first step consists in multiplication by a demodulating factor, namely, a complex exponential at frequency $-\omega$:

$$s(t)e^{-j\omega t} = x(t) \quad (4)$$

Multiplication thus brings the signal to baseband. After that, a low-pass filter is used. In particular, time averaging (coherent integration) can be employed.

In summary, double demodulation consists of a "normal" quadrature demodulation, usually obtained in an analog fashion, followed by a second demodulation, performed on the complex signal obtained from the first one. This latter operation, in turn, consists of a multiplication followed by coherent integration. A slowly varying complex signal is ob-

equivalent to four real multiplications and two real sums. The latter can be postponed and performed after time averaging to reduce computations. Algorithm implementation would then consist of calculating four time series by real multiplication:

$$A(t) = a(t) \cos \omega t \quad (7)$$

$$B(t) = a(t) \sin \omega t \quad (8)$$

$$C(t) = b(t) \cos \omega t \quad (9)$$

$$D(t) = b(t) \sin \omega t \quad (10)$$

and integrating (accumulating) them separately the required number of times.

After some algebraic manipulation, the baseband components of the observed spectral line can be obtained as

$$c(t) = \sum A(t) + \sum D(t) \quad (11)$$

$$d(t) = \sum C(t) - \sum B(t) \quad (12)$$

where \sum is the time averaging (integration) operator.

It is interesting to note that the above operations allow observation of two symmetrical spectral bands at the same time, because baseband signals for spec-

tained, the spectrum of which reproduces the shape of the line under observation. We remark that no distortion is introduced by the multiplication, because a pure complex exponential is used as the demodulating factor.

The same algorithm can of course be applied for demodulating a spectral line at $-\omega$ (i.e., a negative Doppler shift), by using $e^{j\omega t}$ as the demodulating factor. If two Doppler spectral lines are present at both frequencies ω and $-\omega$, however, care must be taken to avoid aliasing. If we want to observe the line at $+\omega$, the demodulation factor is $e^{-j\omega t}$. The band around $-\omega$ is shifted to -2ω : The low-pass filter should efficiently attenuate such a frequency to avoid significant aliasing.

To see how the algorithm is implemented, let us consider the complex multiplication in some more detail. The terms $s(t)$ and $x(t)$ can be expressed as

$$x(t) = c(t) + jd(t) \quad (5)$$

$$s(t) = a(t) + jb(t) \quad (6)$$

where $a(t)$ and $b(t)$ are the in-phase and in-quadrature receiver outputs. The complex product in (4) is

at the same time, because baseband signals for spectral lines at $+\omega$ and $-\omega$ can both be calculated from $A(t)$, $B(t)$, $C(t)$, and $D(t)$. In fact, if two spectral lines are present in the signal

$$s(t) = x(t)e^{j\omega t} + y(t)e^{-j\omega t} \quad (13)$$

with

$$y(t) = c'(t) + jd'(t), \quad (14)$$

it can be shown that $c'(t)$ and $d'(t)$ can be obtained as

$$c'(t) = \sum A(t) - \sum D(t) \quad (15)$$

$$d'(t) = \sum C(t) + \sum B(t) \quad (16)$$

The complex time series $x(t)$ and $y(t)$ can then be separately processed to evaluate the spectra of $+\omega$ and $-\omega$ bands.

As a further advantage of the above approach, we remark that the processing system can be designed to store also the initial $s(t)$ values. By means of plain coherent integration, the spectral band around zero frequency can be observed in parallel to the other two.

4. Double Demodulation by Sign Reversal

The main difficulty in implementing the above technique in a real-time processing system is the presence of real multiplications. Values for the sinusoids would have to be calculated or extracted from a table, and data would have to be multiplied and accumulated in real time. The processing system may be too slow to perform these operations in real time, especially when many heights have to be processed in parallel.

If available processing power doesn't allow an implementation of the complete algorithm with sine wave multiplication, a simplified version of it can be used, in which sine waves are replaced by square waves. Such simplification is obtained at the cost of some distortion of the signal, as we are now going to discuss.

The key point is the following: Multiplication by a sine wave is approximated by an alternated sign reversal operation. The signal to be demodulated is multiplied by $+1$ or -1 according to the sign of the demodulating sine wave. The process is performed in parallel for the $\cos \omega t$ and $\sin \omega t$ components. In other words, the $\sin \omega t$ and $\cos \omega t$ functions are replaced by a couple of square waves 90° out of phase. The demodulation and integration process is reduced to adding data samples into separate accumulators, reversing their sign when the corresponding square wave is -1 and leaving them unaffected when it is $+1$. These operations are clearly of faster execution than floating point multiplications. After such sign reversal, coherent integration would be used. We are going to consider separately the effects of such operations on spectral lines, if present, and on white noise.

Consider the complex signal defined as follows:

$$h(t) = Sq(\cos \omega t) + jSq(\sin \omega t) \quad (17)$$

where $Sq(\)$ is a modified signum function, which only takes values $+1$ and -1 :

$$\begin{aligned} Sq(x) &= 1 & x \geq 0 \\ Sq(x) &= -1 & x < 0 \end{aligned} \quad (18)$$

Here $h(t)$ is the "square wave" version of a complex exponential, in which the real and imaginary part are square waves 90° out of phase. We call $h(t)$ a "complex square wave" of frequency ω , and we are going to use it as a demodulating signal. Some calculations show that $h(t)$ can be expanded in a Fourier as

$$h(t) = \sum_{n=-\infty}^{+\infty} \frac{8}{4n+1} e^{j(4n+1)\omega t} \quad (19)$$

The spectrum is thus composed of a fundamental frequency ω and of harmonics at frequencies $\omega \pm 4n\omega$. Therefore harmonics are present at $5, 9, 13, \dots$, and at $-3, -7, -11, \dots$, times the fundamental frequency. No harmonic is present at $-\omega$, as we expect, because $h(t)$ is the result of a positive fundamental frequency.

The receiver output signal is thus sampled and multiplied by a sampled version of the complex square wave $h(t)$. Such "discrete time" multiplication is equivalent to analog multiplication followed by sampling, so we consider the latter operation sequence. In the frequency domain the Fourier transform of the signal is convoluted with that of the complex square wave. As a result, the spectral line at radian frequency ω is lowered at zero frequency, and smaller-amplitude copies of it appear at $\pm 4\omega, \pm 8\omega$, and so on. We will refer to such copies as to "ghost lines."

If some spectral feature were present at zero frequency, it would be shifted to $-\omega$, with ghosts at $+3\omega, +7\omega, \dots$, and at $-5\omega, -9\omega$, etc. Also, if a spectral line were present at $-\omega$, it would be lowered to -2ω , with ghosts at $+2\omega, \pm 6\omega, \pm 10\omega$, and so on. Aliasing of one of the ghost lines into zero frequency, and hence signal distortion, can take place if the original signal has any spectral features at frequencies $\omega \pm 4n\omega$, i.e., at $5, 9, \dots$, or $-3, -7, \dots$, times the square wave frequency. Special care must be taken to evaluate this possibility case by case. A digital anti-aliasing filter (e.g., a low-pass with cutoff ω) could be used to prevent this problem, but such a solution can imply more computational effort than required by plain multiplication by sine waves. We now imagine that the signal is sampled after multiplication: As a consequence, a frequency-domain "folding" takes place. Spectral features that lie outside the Nyquist band are folded and appear as aliases in the Nyquist band. The effect can be visualized as the following: Each spectral feature is moved in constant steps until it falls within the observed band, where it adds up to the complex spectrum.

In particular, the ghost lines generated by multiplication are folded back into the observed band and can give rise to aliasing. Depending on the relationship between ω and the (radian) sampling frequency, ω_s , the following cases can occur:

1. If ω_s is a multiple of 4ω , each aliased ghost line is coincident with another copy of itself, because such copies were separated by 4ω . The net effect is that the shape of each spectral line is undistorted. Furthermore, the total effect on each line can be calculated

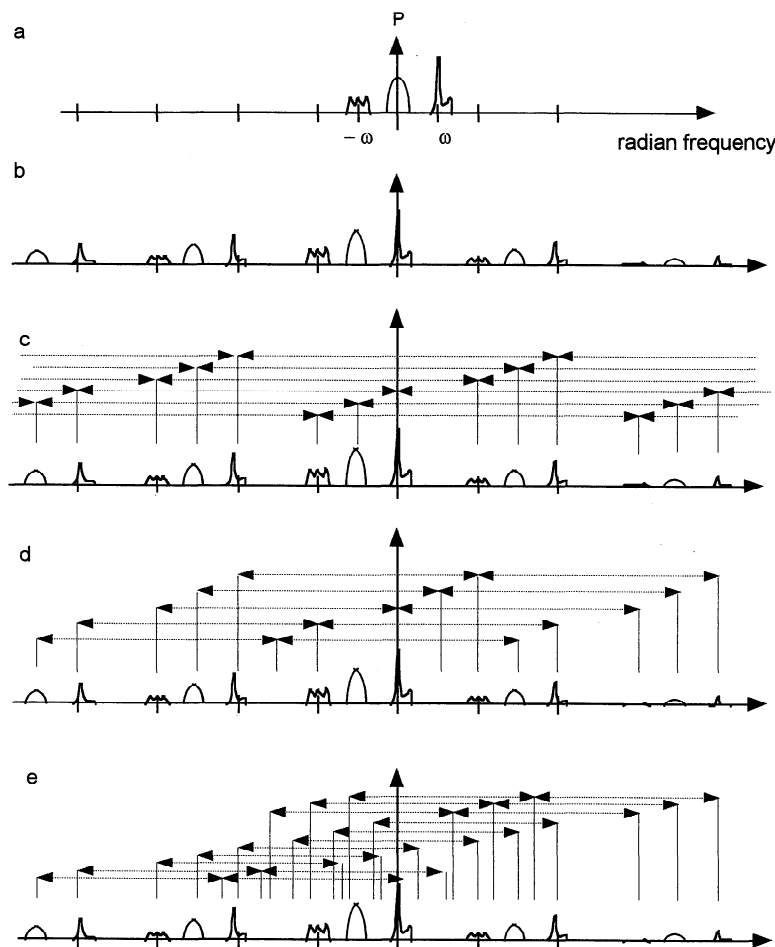


Figure 1. The effects of time-discrete multiplication by a complex square wave can be visualized considering a continuous-time multiplication followed by sampling. Let us consider a signal like in Figure 1a, which has spectral features at frequencies ω and $-\omega$, as well as at zero frequency. Continuous-time multiplication by a square wave of frequency ω causes an infinite set of "ghost lines" to appear (Figure 1b). Sampling can have different effects. If the sampling frequency is an integer multiple of 4 times the square wave frequency, each line is coincident with another copy of itself, and no net aliasing takes place (Figure 1c); if the sampling frequency is a multiple of ω but not of 4ω , or if it is not a multiple of ω , the spectrum is distorted due to aliasing (Figures 1d and 1e).

because the amplitude of each ghost line is known (proportional to $1/(4n + 1)$, as shown before).

2. If ω_s is a multiple of ω but not of 4ω , ghosts of the spectral lines which in the original signal corresponded at frequencies zero and $-\omega$ can be aliased into zero frequency and distort the observed signal.

3. If ω_s is not a multiple of ω , aliased ghost lines can spread over all the frequency field of interest, and due to the slow decay of the $1/(4n + 1)$ term, strong distortion can take place.

The conclusion is that sampling frequency ω_s should be a multiple of 4ω for minimum distortion.

Figure 1 shows a representation of the effects of multiplication on the spectral content of a signal in the above cases. We note that an even better situation occurs when ω_s is exactly 4ω . In this case the original algorithm can be implemented with small effort, because the sine wave samples reduce to $\sin n\omega t_s = 0, 1, 0, -1, 0, 1, 0, \dots$, $\cos n\omega t_s = 1, 0, -1, 0, 1, 0, -1, \dots$, so that multiplications by $+1$ and -1 are sufficient. This, however, is a particular case.

After sign reversal, the signal is coherently integrated to select the spectral line near zero frequency. The observed band is reduced by a factor equal to the

number of coherent integrations, N . In general, a frequency range much narrower than ω will be observed.

The signal spectrum before coherent integration contains a number of ghosts of the zero-frequency spectral line at $\pm 4\omega$. Further sets of ghost lines are present at $(-1 \pm 4n)\omega$ and $\pm 2n\omega$ if $s(t)$ had features at zero frequency and at $-\omega$. All such lines can cause aliasing, and further distortions, because of the decimation implied in the coherent integration process. There is no way to eliminate these lines completely by coherent integration. The problem can be reduced by considering that the first step in coherent integration is a boxcar filter with transmission zeroes at radian frequencies $n(\omega_s/N)$. If ω is a multiple of ω_s/N , all spectral lines except the one at zero frequency fall on transmission zeroes, and all ghost lines are strongly attenuated. The conclusion is that for minimum distortion, the coherent integration period should be made a multiple of the square wave period.

All the above discussion holds for spectral lines. White noise is also present in virtually every radar signal, and it is important to evaluate the effects of the proposed algorithm on it. This is very easily done by considering that white noise is uncorrelated from sample to sample. Also, it can have positive or negative values with equal probability. Hence sign reversal has no effect on noise correlation, because it produces another uncorrelated time series with the same statistics. The effects on noise of the whole process are thus the same as for plain coherent integration.

With careful evaluation of distortion and aliasing effects, this simplified procedure makes double demodulation affordable even for real-time processing systems. Some problems may arise to fulfill the constraints we introduced for minimum distortion, i.e., that the sampling period, square wave period, and coherent integration period be multiples of each other. Such problems can usually be circumvented in existing radar systems. In fact, (1) the sampling period is equal to the radar IPP (interpulse period, i.e., the time interval between successive transmitted pulses); it is usually possible to increase the IPP, at the cost of a reduction of the mean transmitted power, to adjust it to a submultiple of the square wave period. On the contrary, it is not usually possible to reduce the IPP, because multiple-time-around echoes can appear; (2) on the other hand, there is no need to use a square wave frequency strictly equal to the spectral line frequency: there can be a small difference, because the algorithm allows observation of a small frequency band; and (3) spectral lines are usually narrow, and

rather large values of N can be used, so it is not difficult to find a coherent integration number multiple of the required factors.

A typical parameter selection procedure can thus be the following: (1) determine the spectral line frequency to observe; (2) determine a square wave frequency F_q close to the spectral line; if necessary, adjust the IPP so that sampling frequency is a multiple of $4F_q$; and (3) determine the coherent integration period as the largest multiple of the square wave period that allows observation of the required bandwidth.

5. Possible Implementations

A possible hardware implementation of double demodulation is shown in Figure 2. We consider that an integer multiple of the sampling period is used for the square wave. We further suppose that the phase of the square waves and the sample pulses are synchronized so that no sample falls on the square wave transition front. The easiest way to obtain this is to choose a square wave period multiple of 4 times the sample rate: This way, the 90° phase shift between "sin" and "cos" square waves contains an integer number of samples. Thus each square wave period is divided into four "quadrants" where the two control signals are (1, 1), (-1, 1), (-1, -1), and (1, -1), respectively. These control signals can be generated by a two-bit register, cycling through four states, clocked by a simple programmable counter. If a square wave period contains $4N$ samples, the programmable counter generates a pulse every N samples, which triggers a transition in the register state. The register bits in turn control two programmable sign switchers. If nonmultiple periods are used, control pulses have to be generated by a separate oscillator, possibly linked to the system clock. Another possible implementation, which avoids sign switchers, is to use the control signals as address bits to select the accumulator where samples will be added. So all samples corresponding to a +1 control signal are accumulated in one register, and all samples corresponding to -1 are accumulated in another. At the end of the coherent integration period the content of the second register is subtracted from the first; the result is the same as obtained by a sum with sign switching. This solution avoids sign switching but requires the use of eight accumulators: one accumulator is needed for each state (+1/-1) of each of demodulating signal (sin and cos) for each of the components (real and imaginary) of the input signal. A software implementation is also

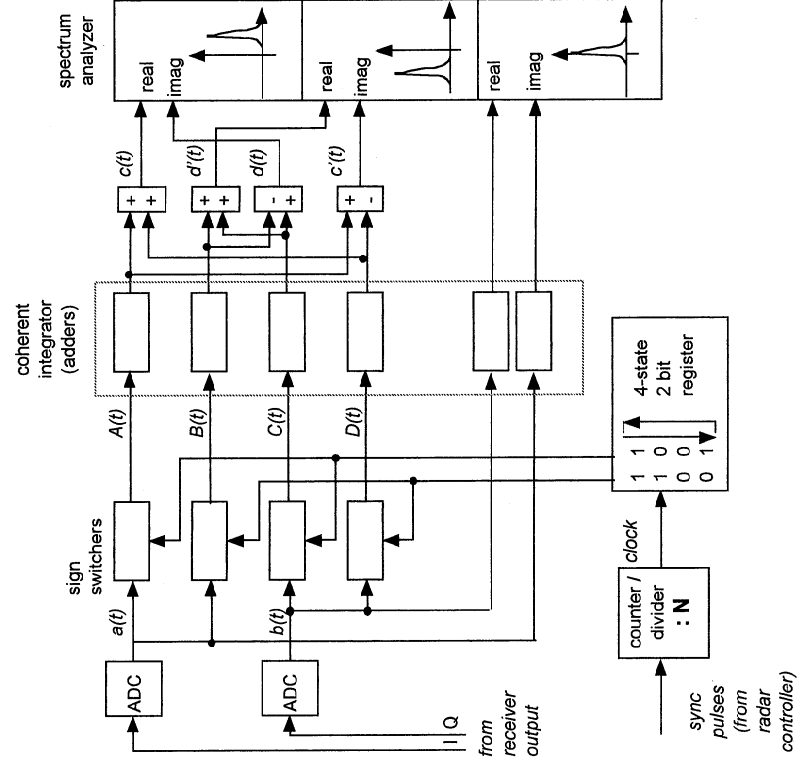


Figure 2. A possible hardware implementation of double demodulation using complex square waves.

possible, provided a fast digital signal processor (DSP) is used. The program would simply generate the averaged values by accumulating N samples and subtracting the following N . If the DSP is fast enough, real multiplication by sine waves may be implemented. A DSP solution may be facilitated by the fact that as explained above, four independent accumulators are used for the four results of multiplications (or sign reversal). A pipelined architecture could efficiently perform the four accumulations, because each accumulation result will only be needed after three other operations. As explained before, accumulator contents would then be added together after the specified number of integrations has been performed.

6. Conclusion

The double demodulation technique we propose is a digital reposition of the analog operations performed by radar receivers to obtain baseband signals. Such an algorithm will probably find application in Doppler experiments where narrow spectral lines are measured, like radio-acoustic sounding or sea scatter experiments. The main advantage of double demodula-

tion consists of a reduction of computational load when high-resolution frequency spectra have to be estimated. Also, it would be easy to implement such a technique, by hardware or software means, with slight modifications to existing Doppler signal processing systems.

References

- Kcecler, R. J., and R. E. Passarelli, Signal processing for atmospheric radars, in *Radars in Meteorology*, edited by D. Atlas, pp. 199–229, Am. Meteorol. Soc., Boston, Mass., 1990.
- Rastogi, P. K., Spectral and correlation analysis with applications to middle-atmosphere radars, in *Handbook for Middle Atmosphere Program (MAP)*, vol. 30, edited by S. Fukao, Univ. of Ill., Urbana, 1989.
- E. Ragaini, Dipartimento di Elettrotecnica, Politecnico di Milano, Piazza Leonardo da Vinci 32, I-20133 Milano, Italy. (e-mail: ragaini@bottani.etc.polimi.it)
- R. F. Woodman, Radio Observatory de Jicamarca, Apartado 13-0207, Lima 13, Peru. (e-mail: ron@roj.org.pe)

(Received June 18, 1996; revised November 7, 1996; accepted November 19, 1996.)