AN AUTOCORRELATION SCHEME USING A COMPLEX APPROACH (PHASE DETECTORS) WITH N MULTIPLIER—ACCUMULATORS FOR N FREQUENCY POINTS

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The figure of merit of an autocorrelator when built using a modular approach, is given by the number of multiplier - accumulators and by the speed of its clock. One can build a correlator box for as many lags as desired by putting as many modules together as necessary. The clock speed determines the maximum bandwidth of the signal being processed, and the number of multiplier—accumulators, the number of points in the frequency spectrum, i.e. the frequency resolution of the instrument. A multiplier-accumulator is the basic module of a correlator, it performs the multiplication of the present value with a delayed value of the signal and up-dates the contents of a register where the corresponding time average is stored.

There are two basic approaches to the problem of evaluating the spectrum of a narrow-band signal (also referred to as quasy-sinusoidal process). In one approach, the signal is converted to the lowest IF frequency possible, i.e. to a frequency equal to half the bandwidth of the signal and then it is processed as an ordinary real signal. We shall call this the real autocorrelation approach. The other approach consists of converting the signal into two quadrature signals so that the center frequency goes to zero frequency.

The pair of signals can be conveniently represented as a complex pair. The complex correlation function is, then, evaluated. The Fourier transform of the function is the spectrum of the signal, where zero frequency corresponds to the center frequency of the original process. We shall refer to it as the complex autocorrelation approach.

The advantage of the complex approach is that, for a given clock speed, it can characterize a process with twice the bandwidth of the real approach, and more importantly, it uses a frequency converter much simpler and more economical than the one needed for the real approach. Furthermore, the bandwidth of the instrument is easier to control and achieve, since it involves only low-pass filters rather than sharply defined band-pass filters and single-side-band discriminating schemes. The disadvantage is that it requires twice the number of multipliers and accumulators for the same frequency resolution.

The purpose of this note is to present a scheme which permits implementation of both systems, therefore, allowing one to trade bandwidth for frequency resolution, but using only one frequency conversion scheme, for both: the simpler complex conversion (phase detection) scheme. It involves only the introduction of a time multiplexer between the complex digitized signals and a real correlator.

Let us first introduce the mathematical model of the two approaches. We will then present the proposed approach.

A band-pass limited process, $f_o(t)$, with spectrum $F(\omega;\omega_o)$, such that $F(\omega;\omega_o) = 0$ for $\omega_o - \Delta\omega/2 < \omega > \omega_o + \Delta\omega/2$ can always be represented by two functions $a(t)$ and $b(t)$, such that

$$f_o(t) = a(t)\cos\omega_o t + \sin\omega_o t \qquad (1)$$

where $a(t)$ and $b(t)$ are random processes with frequency spectrum: Even $[F(\omega;0)]$, and therefore band-limited with $F(\omega;0) = 0$ for $|\omega| > \Delta\omega/2$.

The real correlation approach converts $f_o(t)$ into a new process $f_1(t)$ such that

$$f_1(t) = a(t)\cos\omega_1 t + b(t)\sin\omega_1 t \qquad (2)$$

with frequency spectrum $F(\omega;\omega_1)$ where $\omega_1 = \frac{1}{2}\Delta\omega$, and, therefore, band-limited with $F(\omega;\omega_1) = 0$ for $|\omega| > \Delta\omega$.

This approach consists in evaluating the real correlation function of $f_1(t)$, $\rho_1(\tau)$, from which its power spectrum $F(\omega;\omega_1)$ is evaluated. The desired spectrum $F(\omega;\omega_o)$, is obtained by simple substitution of ω_1 by ω_o.

Being a $\rho_1(\tau)$ real correlation, N multiplier-accumulators are needed for N correlation lags. And, since the sampling frequency has to be twice the maximum frequency,

$f_{max} = 2\pi\Delta\omega$, the maximum bandwidth we can process, Δf, is given by

$$\Delta f = \frac{f_k}{2} \tag{3}$$

where f_k is the maximum clock frequency of the module.

The complex correlation approach converts $f_\circ(t)$ into the two signals $a(t)$ and $b(t)$, which we can represent conveniently by a complex signal

$$s(t) = a(t) + jb(t)$$

The complex correlation, $\rho_c(\tau)$, is then evaluated from $\rho_c(\tau) = \langle s(t)s^*(t+\tau)\rangle$. The Fourier transform of which can be shown to be real and positive and given by $F'(\omega)$ related to $F(\omega;\omega_\circ)$ through

$$F(\omega;\omega_\circ) = 1/2 \text{ Even } [F'(\omega;\overset{(\omega+\omega_0)}{\cancel{\omega_\circ}})] \tag{4}$$

The sampling speed in this case is as large as the bandwidth of the signal, $\Delta f = 2\pi\Delta\omega$, since $a(t)$ and $b(t)$ have a maximum frequency content equal to one half the bandwidth of $f(t)$. Therefore, in this case

$$\Delta f = f_k \tag{5}$$

It is a property of the discrete transform, that, N/2 complex lags transform into N real points of the frequency spectrum. The implementation of the complex correlation with standard modules needs 4 multiplier-accumulators for each complex lag, one for each of the four products $\langle a(t)a(t+\tau)\rangle$, $b(t)b(t+\tau)\rangle$, $\langle a(t)b(t+\tau)\rangle$, and $\langle b(t)a(t+\tau)\rangle$. Therefore, 2N accumulators are needed for N points in the power spectrum. But, given a maximum clock rate for the correlator modules, it can process signals with twice the bandwidth of the real scheme. Notice that there is no free lunch; it takes twice as many elements to process twice as much information.

If the correlator is built in four modules capable of being connected in either series or parallel, it would then be possible to use it as either a complex or as a real correlator, trading in the process bandwidth with resolution. But, two different frequency conversion schemes would be needed. And, if maximum resolution is desired, one would have to use the real approach with the complicated single-side-band mixing scheme mentioned before. This is done, for instance, with the Arecibo 1008-channel correlator.

<u>A proposed approach.</u> It is possible to obtain the resolution of the real correlation scheme without the need of a single side-band mixer. For this, it should be noted that after sampling at four times the intermediate frequency (also the proper Nyquist rate), the signal is converted into a sequence $f_1(t_1)$ given by

$$f_i(t_i) = a(t_i) \quad \text{for } i = 0,4,\ldots 4n$$
$$= b(t_i) \quad \text{for } i = 1,5,\ldots 4n+1 \qquad (6)$$
$$= -a(t_i) \quad \text{for } i = 2,6,\ldots 4n+2$$
$$= -b(t_i) \quad \text{for } i = 3,7,\ldots 4n+3$$

where $t_i = 1\frac{2\pi}{4\omega_o}$, that is a sequence of the form

$a(t_o)$, $b(t_1)$, $-a(t_2)$, $-b(t_3)$, $a(t_4)$ ... , etc. Here we have taken the time reference conveniently so that sin $\omega_1 t$ and cos $\omega_1 t$ take a value equal to either one or zero at sampling time.

Our approach is based on the fact that the desired sampled information, $f_1(t_i)$ can be alternatively obtained from the two signals a(t), b(t) obtained in a complex conversion sheme. This can be implemented as shown in Fig. 1, the negative sign of the 4n+2, and 4n+3 samples are taken care of by reversing the sign every two other samples at a logical level in the correlator.

An equivalent way of proving that (5) produces the right spectrum can be obtained by noticing that the different lag accumulators, τ_j, in the correlator correspond to estimates of

$$<a(t_i)a(t_i+\tau_j)+b(t_i)b(t_i+\tau_j)> \quad \text{for } j\text{'s} = 4m$$
$$<a(t_i)b(t_i+\tau_j)-b(t_i)a(t_i+\tau_j)> \quad \text{for } j\text{'s} = 4m+1$$
$$-<a(t_i)a(t_i+\tau_j)+b(t_i)b(t_i+\tau_j> \quad \text{for } j\text{'s} = 4m+2$$
$$-<a(t_i)b(t_i+\tau_j)+b(t_i)a(t_i+t\tau_j)> \quad \text{for } j\text{'s} - 4m+3$$

These values correspond exactly to the values of the real

and the imaginary

part of the complex autocorrelation function $\rho_c(\tau_j)e^{-j\omega_o\tau_j}$ which has Even[F(ω;0)] as its transform.

Since this approach is effectively a way of implementing the real autocorrelation approach defined before; it has the same performance: N frequency values for N multiplier-accumulators and a maximum bandwidth equal to one half the clock velocity. The advantage lies in the fact that it does not require a single-side-band mixer but a much simpler phase detector and filters instead, and that the same detector and filters can be used for either a real or complex approach permitting the user to trade between bandwidth and resolution at will.

An additional advantage, specially important in the case of multiple bit correlator, is that in the real correlation mode, the proposed approach samples the signal at half the speed needed for the conventional real approach. This is important, since in this case the ADC speed would probably be the speed limiting element of the system. Furthermore, the band-pass filter characteristics are guaranteed to be symmetrical; this is an important property when measuring small Dopler shifts from the center frequency, for instance, in incoherent scatter work.

It should be noted that if quadrants (3) and (4), for example, are attached in series with quadrants (1) amd (2), in Fig. 1(6) in the complex configuration mode, it is then possible to get N frequency points with bandwidths equal to the sampling frequency, i.e. the best of all worlds, but at the expense of $\sqrt{2}$ in the value of the statistical deviation of the frequency spectrum estimate. Here we would be using only $\langle a(t)a(t+\tau)\rangle$ and $\langle a(t)b(t+\tau)\rangle$ for the real and imaginary part of $\rho_C(\tau)$, throwing away $\langle b(t)b(t+\tau)\rangle$ and $\langle b(t)a(t+\tau)\rangle$, which have the same value but represent a completely independent estimate.