

**PHASE
MEASUREMENT/DISPLAY SUBSYSTEM
FOR THE
APOLLO RE-ENTRY TRACKING SYSTEM**

DECEMBER 13, 1963



**GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND**

Phase Measurement/Display Subsystem
for the
Apollo Re-entry Tracking System

by

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December 13, 1963

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PHASE MEASUREMENT/DISPLAY SUBSYSTEM

FOR THE

APOLLO RE-ENTRY TRACKING SYSTEM

A combined system consisting of an Interferometer and a Range System has been proposed (Reference #1) for tracking the Apollo spacecraft during the re-entry phase. The system herein discussed processes the output of the above system and performs the following functions (Figure #1):

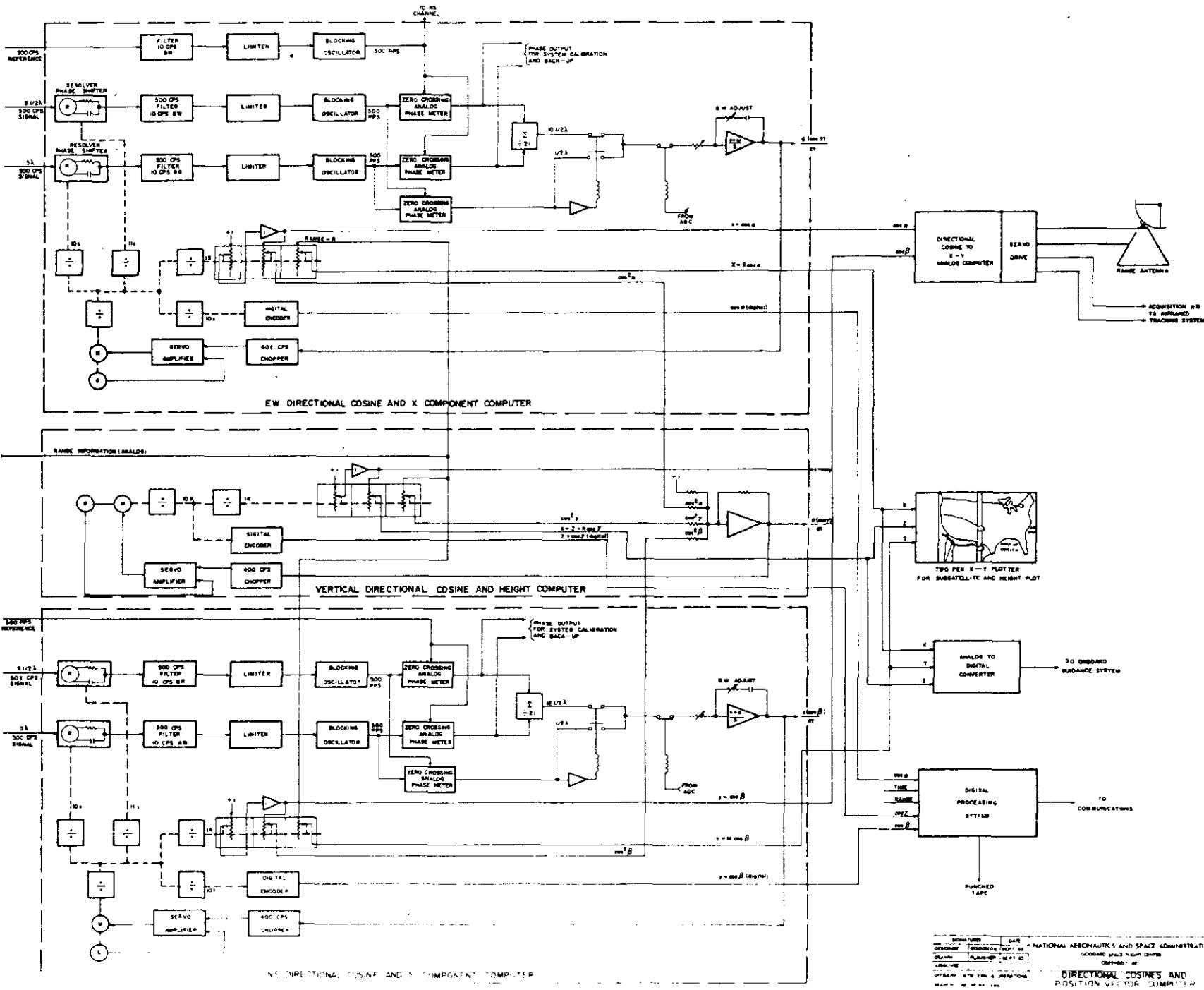
1. Resolves the interferometer ambiguity in real-time.
2. Computes the directional cosines of the tracked object in both digital and analog form.
3. Computes the orthogonal coordinates X, Y, and Z by means of simple analog devices.
4. Plot the sub-satellite position in real-time.
5. Plot the altitude simultaneously with the sub-satellite position.
6. Automatically position the two transmit and receive antennas for the range and telemetry system, the antennas being on an X-Y* mount.
7. Processes and records all needed digital data for transmission to either a central computer to the spacecraft, or both.

The basic idea can be used for other tracking applications with other beacon frequencies and different interferometer baselines.

SYSTEM DESCRIPTION

Interferometer Antenna and Receiver System

The interferometer antenna and receiver system considered in this proposal is the one described by V. Simas (Reference #1). The only change is that baselines of $5-1/2\lambda$ and 5λ are used in both EW and NS direction instead of the



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 TITLE: DIRECTIONAL COSINES AND POSITION VECTOR COMPUTER

proposed $5-1/4 \lambda$ and $4-3/4 \lambda$. These baselines are more convenient for the ambiguity computer. The outputs of the receiver are four 500 cps signals with phase differences with respect to a 500 cps reference signal equal to the phase differences at the four different antennas baselines of the interferometer.

Range Tracking System

The Range system considered in this proposal is a version of the Goddard Range and Range Rate System (Reference #2 and 3). As far as the subsystem herein proposed is concerned, it assumes that the range of the source is available in both analog and digital form.

Directional Cosine Computer

The Directional Cosine Computer takes the 500 cps signals, measures their phase, resolves their ambiguity and converts the information into shaft rotation without undue deterioration of the accuracy inherent in the input signals. The shaft rotation is proportional to the Interferometer phase difference and to the directional cosine in the direction of the Interferometer baseline. It uses phase locked tracking filter techniques having electro-mechanical components. This technique has been used in the "Narrow Band Tracking Filter" (NBTF) developed for the Minitrack System (Reference #4 and 5), the Servo Phasemeter and Analog Ambiguity Computer (SPAAC) (Reference #6), a very similar system to the one proposed herein and originally developed for the Minitrack System, and the Rocket Interferometer Tracking System (RIT). All of them have already been built and are in the process of evaluation.

The phase information is converted to shaft rotation in the following manner (Figure #2). The 500 cps signal with a phase containing the position information is fed to a Resolver Phase Shifter, the rotor shaft of which is servo controlled to keep the output phase constant with respect to the phase of the 500 cps reference. The phase shift produced by the resolver phase shifter is equal to the phase of the input signal. This functional relationship is a continuous one regardless of the magnitude of the phase difference, as the phase increases from 360° to 0° of a new cycle the shaft continuously starts a new revolution. This very important property allows all the analog computations to be performed uniquely.

If one ignores for a moment the ambiguity problem involved, the relationship between the phase difference output of an interferometer, ϕ , and the directional cosine ($\cos \alpha$) is

$$\cos \alpha = \frac{\phi}{N} \quad (1)$$

where

$$N = \text{Antenna Separation in Wavelengths}$$

But

$$\phi = \theta \quad (2)$$

then

$$\theta = N \cos \alpha \quad (3)$$

that is, the angular position θ of the resolver shaft is proportional to the directional cosine. This angular position is measured digitally by a digital shaft encoder. The encoder is geared as close as possible to the resolver to avoid any errors due to gear backlash. The output of the encoder is the digitalized directional cosine, the division by N is accomplished by proper gear ratios and by the electronics of the encoder. An analog output is obtained by gearing down the resolver shaft by $2N$ and placing a precision

potentiometer on the geared down shaft so that when the directional cosine value varies from -1 to 1, the resolver shaft rotates $2N$ revolutions while the potentiometer rotates only once.

The required conformity or linearity of the resolver output shaft vs. electrical phase is determined by the needed tracking accuracy. The required tracking accuracy is 0.1° at 10° elevation which amounts to a change in direction cosine of 0.00031. For a 10 wavelength baseline, the phase must therefore be measured to an accuracy of 0.6%. This presents no problem as resolvers with an accuracy of 0.1% are commercially available.

The method of resolving the ambiguity of each channel is as follows: there are two resolvers in a gear train (for each channel), one is used for the $5\text{-}1/2 \lambda$ baseline channel and it rotates 11 times while the other resolver used for the 5λ baseline rotates 10 times as the directional cosine goes from -1 to +1. For a particular phase of the input signal in the $5\text{-}1/2 \lambda$ channel there are 11 ambiguous positions of the gear train; likewise for the 5λ channel there are 10 ambiguous positions of the gear train. But there is only one position in which both agree. In order to derive this position it is necessary to use a third channel phasemeter besides having one for each of the 5λ and $5\text{-}1/2 \lambda$ channels within the servo-gear train. This third channel phasemeter is known as the $1/2 \lambda$ channel. There is only one unambiguous position of the gear train in which the $1/2 \lambda$ phasemeter produces zero volts output for a given set of 5λ and $5\text{-}1/2 \lambda$ signals. As the gear train deviates to either side from this zero volts position, the output of the $1/2 \lambda$ phasemeter increases monotonically to a positive or negative maximum in the same way that an actual $1/2 \lambda$ baseline channel would. In an actual tracking operation

the $1/2 \lambda$ channel is first used to acquire the target. Once the phasemeter output is very nearly zero, (due to the servo action) the loop control is switched automatically to the sum of both channels which is equivalent to a $10-1/2 \lambda$ channel.

By controlling the gain of the servo loop it is possible to control the effective post-detection bandwidth of the tracking system. Very narrow bandwidths can thus be obtained and this feature has already been exploited in equipment already undergoing field tests. A bandwidth of 0.03 cps has already been achieved. The bandwidth to be used in the Apollo re-entry tracking system can be determined from the expected dynamics and noise level of the signal. For this purpose a two bandwidth system of 3 and .3 cps appears likely. The limitation on the use of a particular bandwidth is the acquisition time, i.e., the transient time of the initial error. The acquisition time, t , is approximately;

$$t = \frac{2.4}{\text{Bandwidth}} \text{ seconds} \quad (4)$$

which corresponds to 0.8 seconds for a bandwidth of 3 cps, and 8 seconds for a 0.3 cps bandwidth.

Position Vector Computer

The X, Y, and Z, components of the source position vector are related to the range, R, and directional cosines, $\cos \alpha$, $\cos \beta$ and $\cos \gamma$, by the following formulae;

$$X = R \cos \alpha \quad (5)$$

$$Y = R \cos \beta \quad (6)$$

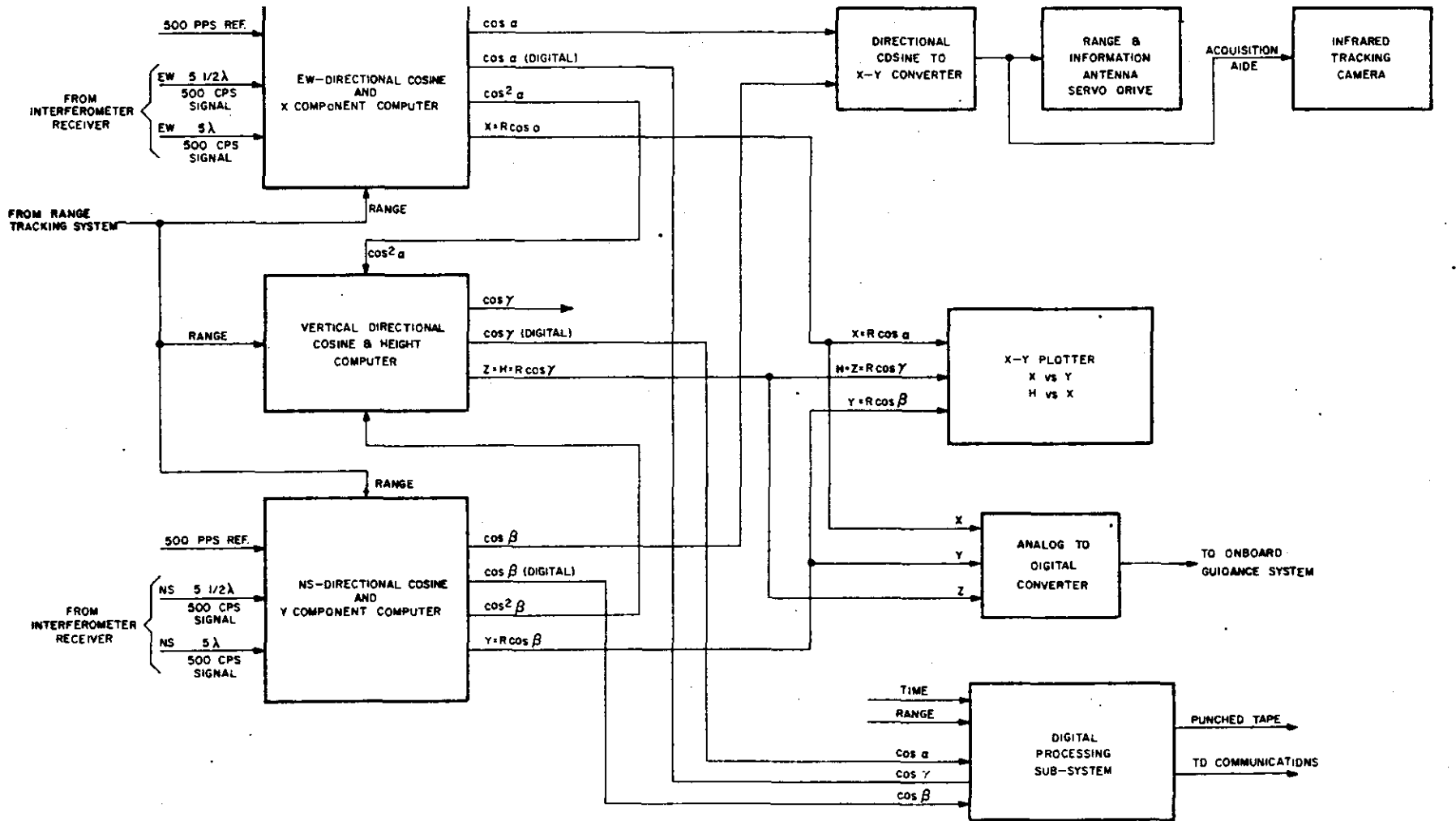
$$Z = R \cos \gamma \quad (7)$$

The first two components X and Y are obtained from the system by simply applying an analog voltage proportional to range to a precision potentiometer placed on the analog directional cosine shaft of the corresponding directional cosine computer (Figure 2 and 3). The potentiometer output is proportional to the voltage applied to it and the position of the wiper, and effectively performs the R times $\cos \alpha$ or $\cos \beta$ multiplication. The accuracy of the multiplication is determined by the linearity of the potentiometer (.1%).

The third component Z (height of the source over the horizontal plane of the station) needs the value of the third directional cosine $\cos \gamma$. $\cos \gamma$ is related to the other directional cosines by

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma - 1 = 0 \quad (8)$$

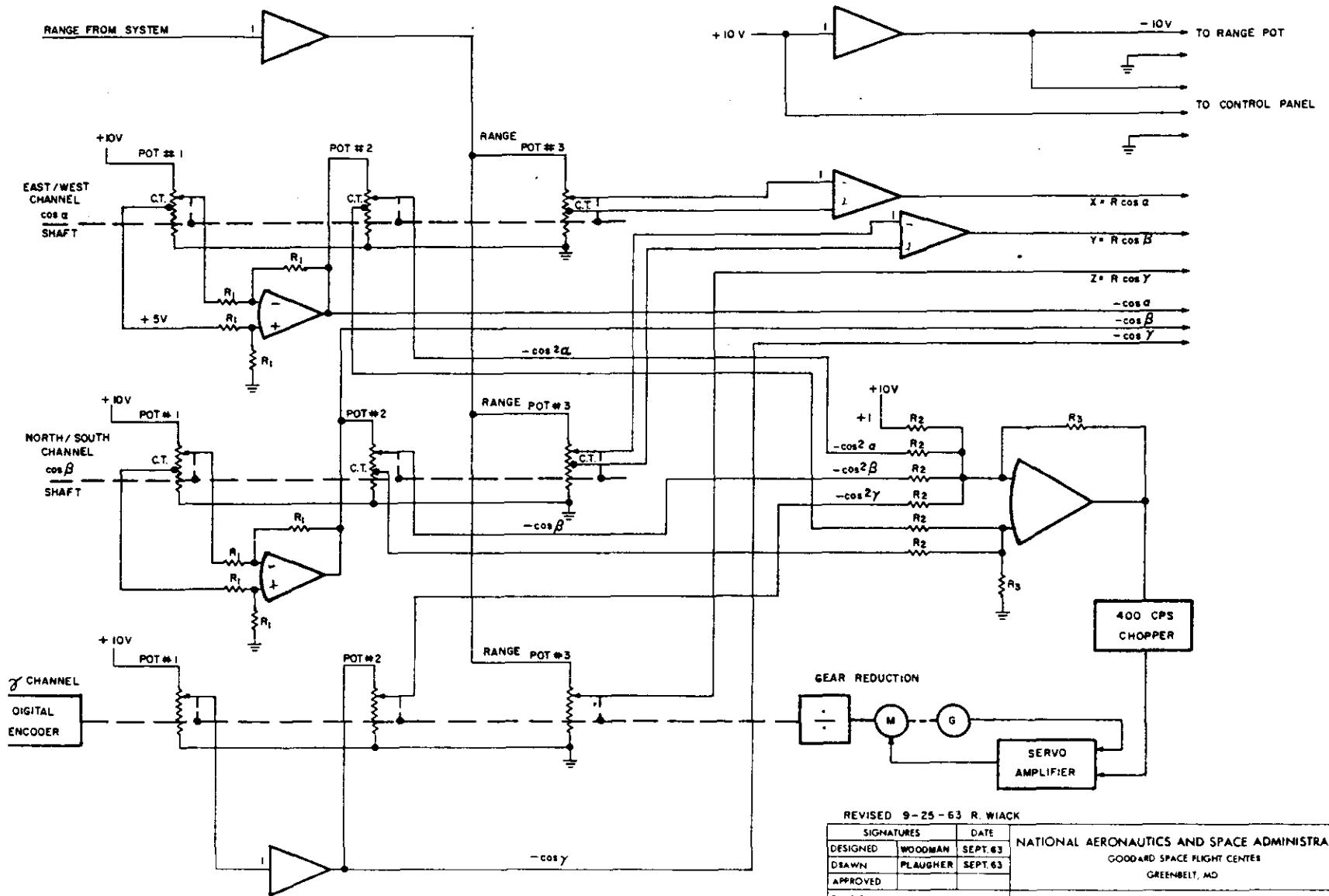
and it is obtained by a servo analog computer. $\cos^2 \alpha$ and $\cos^2 \beta$ are obtained by simply applying the cosine analog output to another pot driven by the same shaft. A third shaft driven by a servomotor is proportional to $\cos \gamma$; $\cos \gamma$ and $\cos^2 \gamma$ analog voltages are generated in a similar way as the other two directional cosines and directional cosines squared. $\cos^2 \alpha$, $\cos^2 \beta$, $\cos^2 \gamma$ and a fixed voltage -1 is fed to a summing amplifier whose output drives the servomotor. If $\cos \gamma$ is not the correct value the output of the operational amplifier is different from zero and drives the servomotor correcting $\cos \gamma$ until the relationship in equation (8) is satisfied. Having $\cos \gamma$ the third position vector component $Z = R \cos \gamma$ is obtained in a similar way as X and Y. $\cos \gamma$ can be digitalized by placing a digital encoder on this shaft. The X, Y, and Z outputs are used for real-time plotting of the spacecraft position. A dual pen x-y recorder is used for this purpose. One pen plots X vs. Y over a geographic map projected on the



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**DIRECTIONAL COSINES AND
 POSITION VECTOR COMPUTER
 FOR A RANGE & INTERFEROMETER
 TRACKING SYSTEM FOR APOLLO
 RE-ENTRY TRACKING**



$R_1 = 10K$
 $R_2 = 10K$
 $R_3 = 10K$

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POSITION VECTOR
ANALOG COMPUTER

horizontal plane of the station. The second pen plots Z vs. X which represents the height of the spacecraft over the horizontal plane. Both plots represent a complete real-time display of the spacecraft position. Tolerances for a nominal trajectory could be plotted on the map prior to the actual track for comparison purposes.

Directional Cosine to X-Y* Converter

One of the direct uses of the directional cosine data obtained from the interferometer system is to drive the directional antennas used to transmit and receive the telemetry information and to transmit and receive the Range System signals. The accuracy requirements are relatively coarse since for the antenna gain specified (19 db) the beamwidth would be in the order of 20° . The system described below should be able to provide a positioning accuracy in the order of a degree of arc or better.

If an X-Y* type of mount is used, a transformation of coordinates is necessary to convert the directional cosines into X-Y* angles. The mathematical relationship between these two sets of coordinates is:

$$\cos \alpha = \sin X^* \tag{9}$$

$$\cos \beta = \sin Y^* \cos X^* \tag{10}$$

This functional relationship is accomplished by the system shown on Figure 4. Two servomotors drive the X* and Y* shaft respectively. A dc voltage proportional to $\sin X^*$ is obtained by means of a functional precision potentiometer (0.1%) placed on the X* axis shaft. This output is compared with the $\cos \alpha$ dc analog output from the interferometer. They should be equal; if they differ, the error drives the X* axis servomotor until the error is reduced to zero and

equation (9) is valid. For the Y^* axis, a dc voltage equal to $\sin Y^*$ is generated by driving a sine functional potentiometer with the Y^* axis, the dc voltage applied to this potentiometer is equal to $\cos X$ and comes from a cosine potentiometer driven by the X^* axis. The $\sin Y \cos X$ dc voltage is compared with the $\cos \beta$ dc analog output from the interferometer system and the error is used to drive the corresponding X^* axis servomotor until it is reduced to zero and relationship (10) holds.

Inaccuracies of 0.1% on the directional cosine voltage correspond to 0.1° error at zenith and errors in the order of 0.5° for 10° elevation angles. Errors due to the functional sine and cosine potentiometers are of the same order. In the worst condition this source of error would add up arithmetically (more probable value would be the square root of the sum of the squares) and errors of the order of 1° for 10° elevation angle and 0.2° at optimum positions should be expected.

Only one converter is necessary for both antennas. Synchro transmitters could be placed on the $X-Y^*$ axis to slave the second antenna. Other similar mounts could be slaved such as an infrared tracking camera for the purpose of acquisition aide prior to radio blackout.

A $X-Y^*$ mount model has been built to perform this conversion and is being evaluated. The idea has been proposed for the Minitrack System with the directional cosine data obtained from SPAAC. In this unit resolvers are used instead of functional sine or cosine potentiometers. Both approaches are equally feasible.

System Data Outputs

Range, $\cos \alpha$ and $\cos \beta$ completely define the position of the source. They are available in digital form with an accuracy of the Range and the Interferometer system. As it has been stated above the accuracy of the transformation from phase difference to shaft rotation and to digitalized directional cosine is of the order of N times the accuracy of the fractional phase difference as measured by the interferometer system or better. The range information accuracy would depend on the side tones used; for 100 kc tones, errors would be in the order of 10 meters (Reference 3) which is 1 part in 10^{-4} for a 100 km range. This set of values, the most accurate outputs of the system, are available for transmission on a real-time basis to the airborne guidance system or to a ground support centralized computer. They can also be recorded on punched paper tape for post facto analysis.

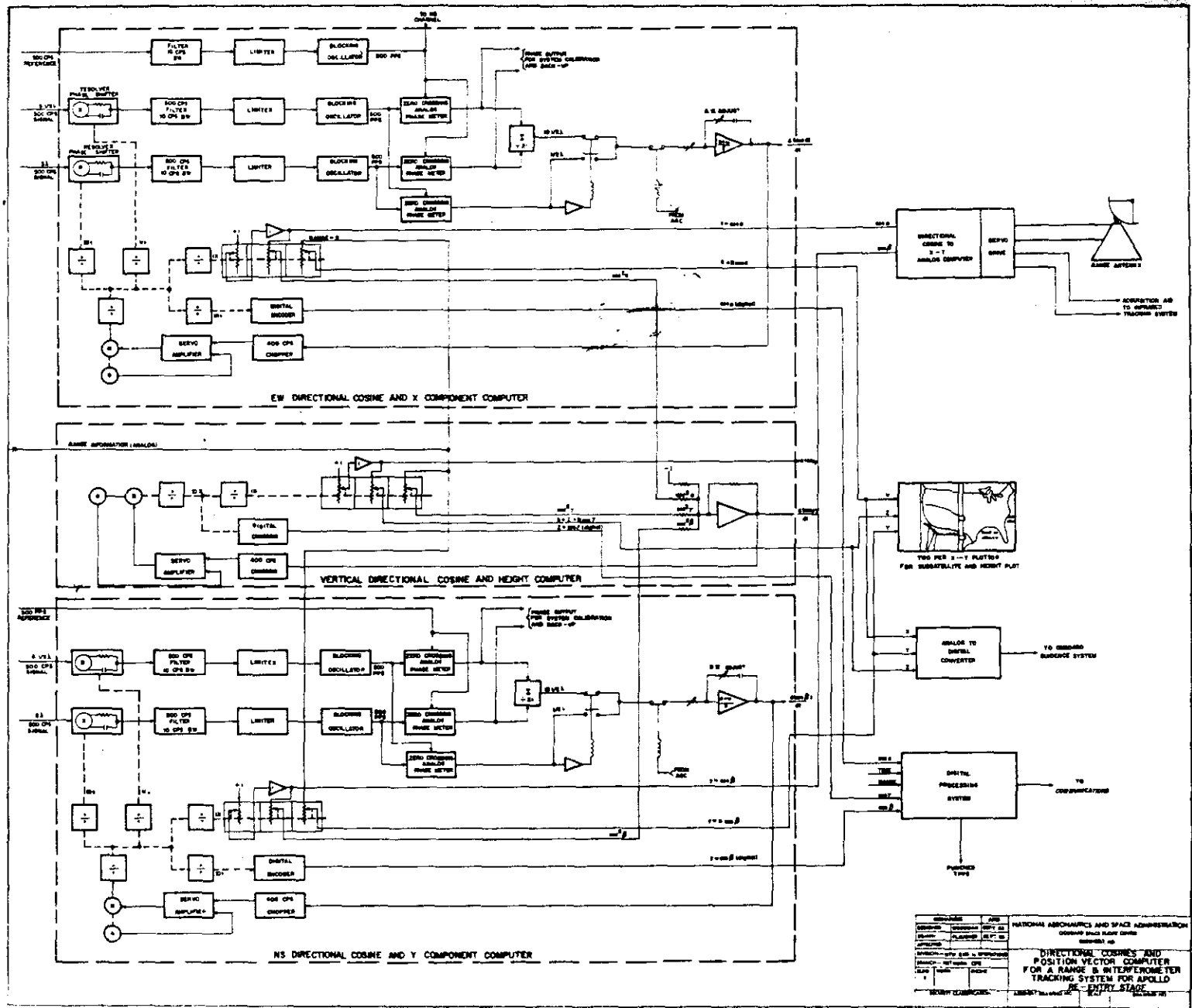
Another set of values that are available from the system are the X, Y and Z components of the position vector. One of the principal uses for this information is the plotting of the subsatellite position and the height on an X-Y recorder as described above. They are not as accurate as the former set of values but accurate enough to meet the tracking requirements (Reference 7). They could also be used for up-dating the on-board guidance system instead of the Z, $\cos \alpha$, $\cos \beta$ set. This advantage is that it is much easier to make computations with the vector components than with any other coordinate system. Translation of coordinates for instance is achieved by arithmetical addition of a constant.

Other outputs available for external use besides the ones already mentioned are $\cos \alpha$, $\cos \beta$, $\cos \gamma$, $\cos^2 \alpha$, $\cos^2 \beta$, $\cos^2 \gamma$, $\frac{d}{dt} (\cos \alpha)$, $\frac{d}{dt} (\cos \beta)$, $\frac{d}{dt} (\cos \gamma)$ all of them in analog form. $\cos \gamma$ is also available in digital form.

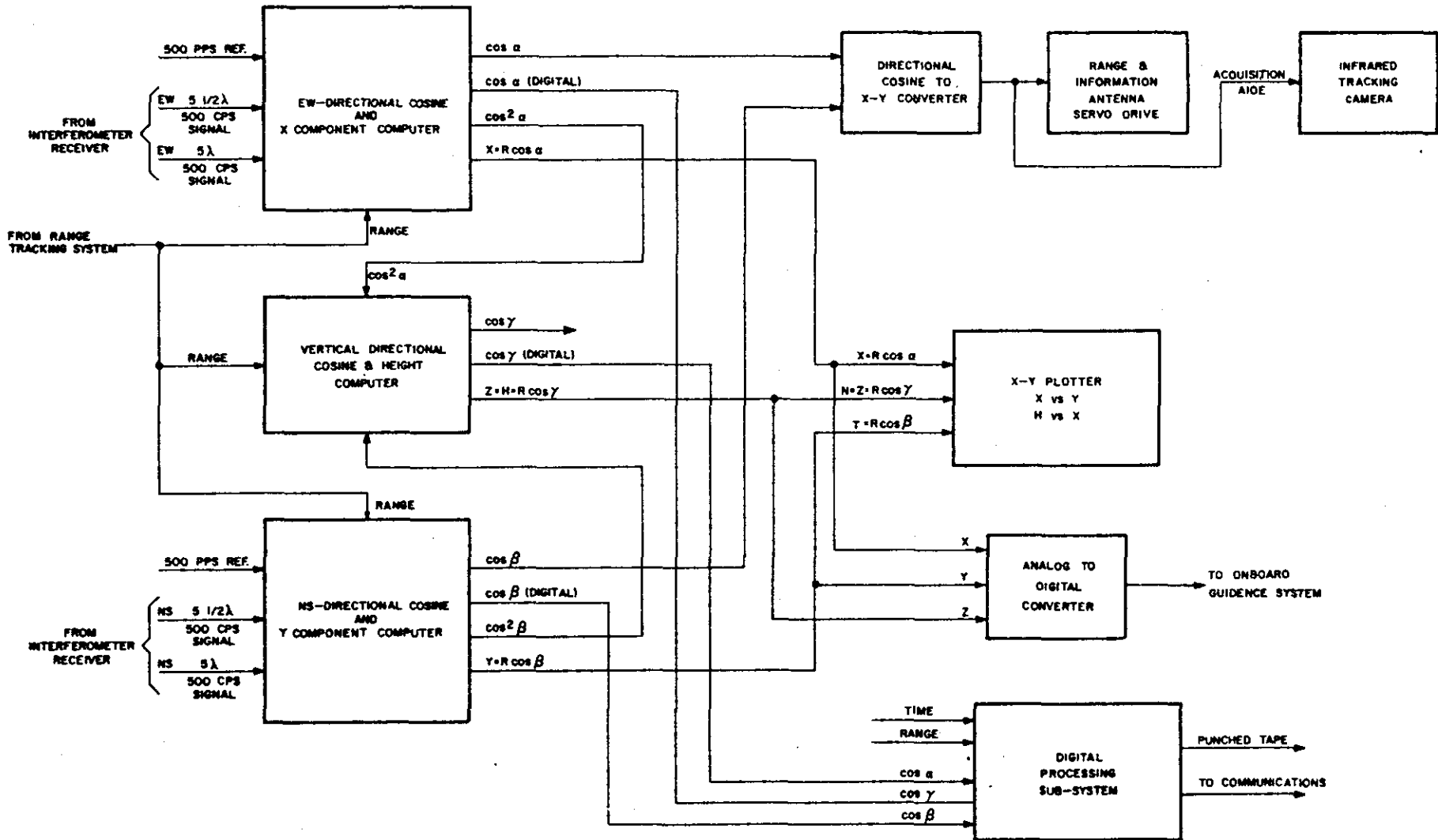
CONCLUSION

A system to meet the requirements for tracking the re-entry stage of the Apollo mission has been described. It uses interferometer information for the angular position without any acquisition problems. It is accurate, simple and relatively inexpensive.

*X-Y refers to the conventional X-Y antenna mount system and it is different from X and Y components of the position vector.



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PROJECT	WORK CENTER	DIRECTIONAL COSINE AND POSITION VECTOR COMPUTER FOR A RANGE & INTERFEROMETER TRACKING SYSTEM FOR APOLLO ENTRY STAGE
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REV. 8	DATE	
REV. 9	DATE	
REV. 10	DATE	



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