In a communication link which uses the scattering properties of either
the neutral atmosphere (tropospheric scattering) or the ionosphere (ionos-
pheric scattering) the signals received are subjected to deep fading.

We can mathematically model the effect of pure scattering by writing
that the signal received, \( y(t) \), from a single frequency transmission, \( f(t) \cos \omega t \)
in the form

\[
 y(t) = A(t) \cos \omega t + B(t) \sin \omega t
\]

where \( A(t) \) and \( B(t) \) are random functions of time with a time scale much slower
than \( \omega^{-1} \). They are Gaussian processes such that

\[
\langle A(t) A(t+\tau) \rangle = \rho(\tau) \langle B(t) B(t+\tau) \rangle = 0,
\]

that is, they are statistically independent of each other, but statistically i-
\[\text{dentical process. They can be completely defined statistically by stating that}\]

\[\text{they are Gaussian processes with autocorrelation } \rho(\tau). \text{ Their autocorrelation}\]

\[\text{function } \rho(\tau) \text{ depends on the dynamics of the medium, more specifically on the}\]

dynamics of that Fourier component of the fluctuations in index of refraction

which is responsible for the scattering.

It is intuitively obvious that there will be a range of frequencies
\( \omega - \Delta \omega < \omega' < \omega + \Delta \omega \) sufficiently close to \( \omega \) for which the effect of the me-
dium is for all practical purposes exactly the same as for the frequency \( \omega \); that
is, we can write that, also:

\[ \gamma(t) = A(t) \cos \omega t + B(t) \sin \omega t \]  \quad (3)

where \( A(t) \) and \( B(t) \) are identically the same functions used in (\ref{eq:2}) provided that \( |\omega' - \omega| < \Delta \omega \). It can be proved that the width \( \Delta \omega \) for \eqref{eq:3} to be true, depends on the geometry of the link and the size of the scattering volume and on the wavelength of the scattered wave. It is not necessary for us to go into their functional relationship here. We shall refer to \( \Delta \omega \) as the correlated bandwidth of the link.

It must be apparent by now that if we are to construct a "black box", so that for an input \( \cos \omega'(t) \) the output is given by \( \gamma(t) \) in (\ref{eq:2}), for any frequency \( \omega' \) such that \( |\omega' - \omega| < \Delta \omega \), then, the box will simulate the effects of the medium on a modulated input with nominal frequency \( \omega \), provided that the bandwidth of the modulated wave is no wider than \( \Delta \omega \). The signal to noise ratio can be simulated by attenuating the signal to the expected level at the receivers.

The purpose of this note is to report on a simple device which has been constructed in our laboratory with the above purpose in mind. The construction of the device was motivated by our interest to see what effect would the equatorial electrojet, as a scattering medium, have on different types of modulation, as well as to study the effect of different demodulation and diversity reception techniques.

Figure 1 shows schematically the device. The balance mixer used is a passive device like the Hewlett-Packard 165A for the random signals, \( A(t) \) and \( B(t) \). We used the sign of the output noise of a receiver, sampled at intervals much shorter than the medium width of the autocorrelation, \( \rho(T) \), that was to be simulated. The spectrum of such a sequence is white within the frequency range of
interest. The sign wave was passed through a low pass filter with an impulse response \( h(t) \), so that \( f(\tau) = \int_{-\infty}^{\infty} h(t) h(t + \tau) dt \). The filter output is close to Gaussian and has the desired autocorrelation function, \( A(t) \) and \( B(t) \).

The statistical independence of both processes can be obtained in different ways. They can be obtained from the same noise source if this is of sufficient bandwidth such that, by sampling and different times, the resulting sign functions are independent. Figure 2 shows a schematic diagram of the random function generators capable of generating simple correlation functions with an exponential shape. With two operational amplifiers in series, and with more sophisticated feedback networks correlation functions with arbitrary shapes could be generated, but it was felt that being able to simulate the width of the autocorrelation function was sufficient for most applications.

The system described in Figures 1 and 2 could be used as elements of more complicated ones. For instance, two of them with two more statistically independent random functions could be used to test the effect of diversity reception. Several of the elementary systems could be placed in parallel at different ranges of the spectrum of the modulated wave, each with a bandwidth tapering at \( \Delta \omega \), to simulate communication systems with a spectrum wider than the correlated bandwidth of the medium.

We have found useful application for the system. It can also be used to generate a signal with a frequency differing from a reference frequency by an arbitrarily slow audio frequency, down to fractions of a c.p.s., with a stability defined by the audio oscillator. This is obtained making \( A(t) \) and \( B(t) \) sinusoidal with frequency, \( \lambda \), at 90° out of phase.
We then obtain

\[ Y'(t) = K \cos \lambda t \cos \omega t - \lambda \sin \lambda t \sin \omega t \]

\[ Y''(t) = K \cos \left[ (\omega + \lambda) t \right] \]

an output frequency exactly \( \lambda \) c.p.s. higher than \( \omega \). A negative value is obtained by interchanging \( A(t) \) with \( B(t) \). Figure 2 shows an input into the system designed to obtain two signals at audiofrequencies and 90° out of phase.

When used in this way the system performs similar to a radio-frequency resolver phase-shifter. It has been used to simulate doppler shifted signals in a radar system to test velocity detection schemes. The system can simulate doppler shifts as small as desired.

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