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LECTURE NOTES

ON

INCOHERENT SCATTER TECHNIQUES

originating from

THE EISCAT WINTER SCHOOL

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(*)

Scattering of E. M. Waves from Dielectric Density Fluctuations

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Radars are used for remote probing of the upper atmosphere. Monstatic and bi-static configurations have been used. The echoes are obtained from the scattering of the illuminating wave by fluctuations in the dielectric properties of the medium under study.

The fluctuations in the local dielectric constant of a medium are direct consequence of fluctuations in the density of the medium or more properly on the density of that component or components in the medium responsable for its dielectric behaviour, e.g. electron density in a ionized gas, "air" density and water vapor in the lower atmosphere, etc.

In the case the medium is in the thermodynamic equilibrium, the fluctuations are reduced to a minimum (thermal level). In such a case, and for a ionized plasma, we refer to the technique as Incoherent Scatter. These fluctuations are never at the zero level due to the discret nature of matter (Summations of delta functions will always produce fluctuations).

Density fluctuations are statistically characterized by the density space-time correlation function $\rho(r, \tau, \underset{\sim}{x})$ define as

$$\rho(\underset{\sim}{r}, \tau; \underset{\sim}{x}) \equiv \langle n(\underset{\sim}{x}, t)\, n(\underset{\sim}{x} + \underset{\sim}{r}, t + \tau)\rangle \qquad (1)$$

where $n(\underset{\sim}{x}, t)$ is the microscopic random density of the medium at position $\underset{\sim}{x}$ in space and time t. In (spatially) homogeneous medium ρ is independent of $\underset{\sim}{x}$ and $\rho(\underset{\sim}{r}, \tau) \equiv \rho(\underset{\sim}{r}, \tau; \underset{\sim}{x})$.

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We shall develop here the functional relationship that exists between the statistical characterization of the signal received in a radar experiment and the fluctuations in the medium characterized by $\rho(\underset{\sim}{r}, \tau; \underset{\sim}{x})$. The fluctuations need not to be at the thermal level, so we are not limited to the incoherent scatter problem. We should point out that the usefulness of large radars for the study of the upper atmosphere is not limited to incoherent scatter. Proof of which is found in the large number of papers produced by the Jicamarca Observatory by studying backscatter echoes from E-region, F-region irregularities and from turbulent fluctuations in the neutral atmosphere. In fact, some smaller radars are built (STARE, SOUSSY and the TS radars) which depend on the enhanced reflectivity produced either by instabilities or turbulence. This could be the case in EISCAT when observing auroral phenomena or the effects of artificial heating. It will also be the case when studying neutral dynamics using backscatter signals from turbulent fluctuations.

Said functional relationship can be found in the literature but it is usually derived under very simplified conditions with assumptions which are not necessarily valid. The derivation is usually heuristic and in many cases difficult to assess the range of validity of the derived expressions. Such approach is, of course, useful for didactic purposes and when the purpose of the paper is on other aspects of the problem. Derived expressions in the literature are usually derived for a specific technique (out of the many described here by Farley) and for specific conditions (e.g. homogeneous media, continuous

The medium under study is illuminated by a e.m. wave of frequency w_o, modulated by an arbitrary complex signal p(t), scattered e.m. waves are received at a different location (or at same as a particular case), coherently detected, properly filtered and decoded (if necessary). We are interested in evaluating the complex autocorrelation of the signal received, O(t), i.e. :

$$C(\tau,t) \equiv \langle O(t)\; O^*(t+\tau)\rangle \tag{2}$$

in terms of the space and time density correlation of the medium.

The signal O(t) is a random process, usually non-stationary, is fully characterized by its time autocorrelation function $C(\tau, t)$. The dependance on t can normally be associated with a given range, h, corresponding to the delay.

We assume: (1) that there is only primary scattering (first Born approximation valid), i.e. the medium is transparent, the illuminating field at a point $\underset{\sim}{x}$ within the medium is due to the primary illuminating field and the scattered fields at $\underset{\sim}{x}$ are negligible; (2) the system is linear, i.e. if $O_1(t)$ is received for $\rho_1(t)$ and $O_2(t)$ for $\rho_2(t)$. Then $\alpha O_1(t) + \beta\, O_2(t)$ is received for a exitation $\alpha p_1(t) + \beta\, p_2(t)$. The linearity of the propagation in the medium are guaranty by the linearity of Maxwell equations.

The linearity of the system allows us to evaluate the output signal as the linear superposition of the contributions of each differential volume, $d^3\underset{\sim}{x}$ with density $n(\underset{\sim}{x},t)$. This differential contribution can be evaluated in terms of the linear operators depicted in figure 2.

Here we have modelled the propagation of the transmitter to the scattering point by a delay operator with delay $T_1(\underset{\sim}{x})$ and an amplitude factor $K_1(\underset{\sim}{x})$ which represent the effect of antenna gain and other system parameters. The scattered signal is proportional to the local instantaneous (random) density $n(\underset{\sim}{x}, t)$ of the medium times the volume $d^3\underset{\sim}{x}$. The dielectric properties of the medium the receiver, antenna, and other propagation properties are contained in a constant gain (in time) $K_2(\underset{\sim}{x})$. There is a delay block with delay $T_2(\underset{\sim}{x})$, a detector and a filter before we finally get our output from the differential contribution from $n(\underset{\sim}{x}, t)$. The filter is characterized by the complex input response $h(t)$ and includes any decoding scheme. Decoding is a convolution operation and can be considered as part of the filter.

The evaluation of the delay functions $T_1(\underset{\sim}{x})$, $T_2(\underset{\sim}{x})$ and the constant terms $K_1(x)$,$K_2(x)$ does not concern us here and are assumed to be known. The output of the system can then be written as

$$o(t,\underset{\sim}{x})d^3\underset{\sim}{x} = d^3\underset{\sim}{x} \int dt' \, K(\underset{\sim}{x}) \, p(t' - T(\underset{\sim}{x})) \, e^{-i\underset{\sim}{w}_o T(x)} \, n(\underset{\sim}{x}, t' - T_2(\underset{\sim}{x})) \, h(t - t')$$

where we have already operated on the "signal" with the delay operators $\delta(t - T_1(\underset{\sim}{x}))$ and $\delta(t - T_2(\underset{\sim}{x}))$. Here we have used $T(\underset{\sim}{x}) = T_1(\underset{\sim}{x}) + T_2(\underset{\sim}{x})$ for the total delay and $K(\underset{\sim}{x}) = K_1(\underset{\sim}{x}) \cdot K_2(\underset{\sim}{x})$. The total signal output is then

$$O(t) = \int d^3\underset{\sim}{x} \; o(t,x) \qquad (4)$$

Also, the difference in propagation time $T_2(\underset{\sim}{x}) - T_2(\underset{\sim}{x} + \underset{\sim}{r})$ is of the order of r_c/c for points within a correlated volumne. This is much smaller than the characteristic time of the decay of the correlation function unless one is dealing with relativistic plasma. Therefore we can ignore this term in the time argument of the correlation function. In addition, the oscillatory nature of the exponential, with a wavelength comparable to the wavelength of the probing wave,makes the integrant unsensitive to any possible long scale structure of the correlation function across the surfaces of constant T.

Furthermore, the almost linear behaviour of $T(\underset{\sim}{x} + \underset{\sim}{r})$ on $\underset{\sim}{r}$ for $|\underset{\sim}{r}| < r_c$ allows us to linearly expand $T(\underset{\sim}{x} + \underset{\sim}{r})$ in the exponent around $\underset{\sim}{x}$ and write:

$$\begin{aligned} w_o T(\underset{\sim}{x} + \underset{\sim}{r}) &= w_o\ T(\underset{\sim}{x}) + w_o\ \nabla_{\underset{\sim}{r}}\ T(\underset{\sim}{x}) \cdot \underset{\sim}{r} \\ &= w_o\ T(\underset{\sim}{x}) + k(\underset{\sim}{x}) \cdot \underset{\sim}{r} \end{aligned} \tag{7}$$

where $\underset{\sim}{k}(x) = \underset{\sim}{k}_1(\underset{\sim}{x}) - \underset{\sim}{k}_2(\underset{\sim}{x})$, and $\underset{\sim}{k}_1(\underset{\sim}{x})$ and $\underset{\sim}{k}_2(\underset{\sim}{x})$ are the local wave number of the incident and scattered wave, respectively. With this approximations we can write:

$$\begin{aligned} C(\tau,t) = \int & d^3x\ dt'd\tau'\ K^2(\underset{\sim}{x})\ p(t' - T(\underset{\sim}{x}))\ p^*(t' + \tau' - T(\underset{\sim}{x})) \cdot \\ & h(t - t')\ h^*(t + \tau - t' - \tau')\ \hat{\rho}\ (\underset{\sim}{k}(\underset{\sim}{x}),\tau';\ \underset{\sim}{x}) \end{aligned} \tag{8}$$

For homogenous media and constant $\underset{\sim}{k}(\underset{\sim}{x}) = \underset{\sim}{k}$, the spatial integral is independent of ρ and defines a volume, V, and we have

$$C(\tau) = a^2K^2\ V \int \hat{\rho}(\underset{\sim}{k},\tau)\ \phi_{hh}\ (\tau - \tau')d\tau' \tag{11}$$

Above equations, if expressed in the frequency domain, take a even simpler form where the convolution integral is transform to a product of frequency functions.

Case 2. Filter time scale smaller than characteristic time of ρ .

In this case the integrant is different from zero for small values of the argment of $h(\cdot)$, i.e. when

$$t \simeq t'$$

$$\tau \simeq \tau' + t' - t$$

Thus, $\rho(\underset{\sim}{k}(\underset{\sim}{x}),\tau';x)$ can be taken out of the τ' integral evaluated at $\tau' = \tau$. We can then write (8) as

$$C(\tau,t) = \int d^3\underset{\sim}{x}\ \ K^2(\underset{\sim}{x})\ \ \hat{\rho}(\underset{\sim}{k}(\underset{\sim}{x}),\tau;\underset{\sim}{x})\ \bar{p}(t-T(\underset{\sim}{x}))\,\bar{p}^*(t + \tau - T(\underset{\sim}{x})) \tag{12}$$

where $\bar{p}$ is defined as

$$\bar{p}(t) = \int dt'p(t')h(t-t') \tag{13}$$

corresponds to the particular delay t of the measurement. Therefore we will write $\rho_t(k,\tau')$ to extend the generality.

We can also perform the spatial integral in terms of the variables $\underset{\sim}{s}$ and T. Only $K^2(\underset{\sim}{x})$ is a function of $\underset{\sim}{s}$ and we can perform the integral with respect to this variable. If K^2 is a factor which groups all the dimensional factors in $K^2(\underset{\sim}{x})$ then the spatial integral gives us $K^2A(T)$, where A(T) is a equivalent area defined by the $\underset{\sim}{s}$ dependence of the beam pattern. On most cases of interest A(T) is a slow varing function of T, slower than the pulse length and can be taken out of the integral evaluated at the sampling delay t. Considering above we write equation (8) as

$$C(\tau,t) = CK^2A(t)\int d\tau' dt' dT\ \hat{\rho}_t(\underset{\sim}{k},\tau')\, p(t'-T)\, p^*(t'+\tau' -T)\ h(t-t')h(t+\tau-\tau'-t')$$

$$= CK^2A(t)\int d\tau' \rho_t(k,\tau') \int dt'\ h(t-t')\ h^*(t-t'+\tau-\tau')\int dT\ p(t'-T)p^*(t'+\tau'-T) \qquad (16)$$

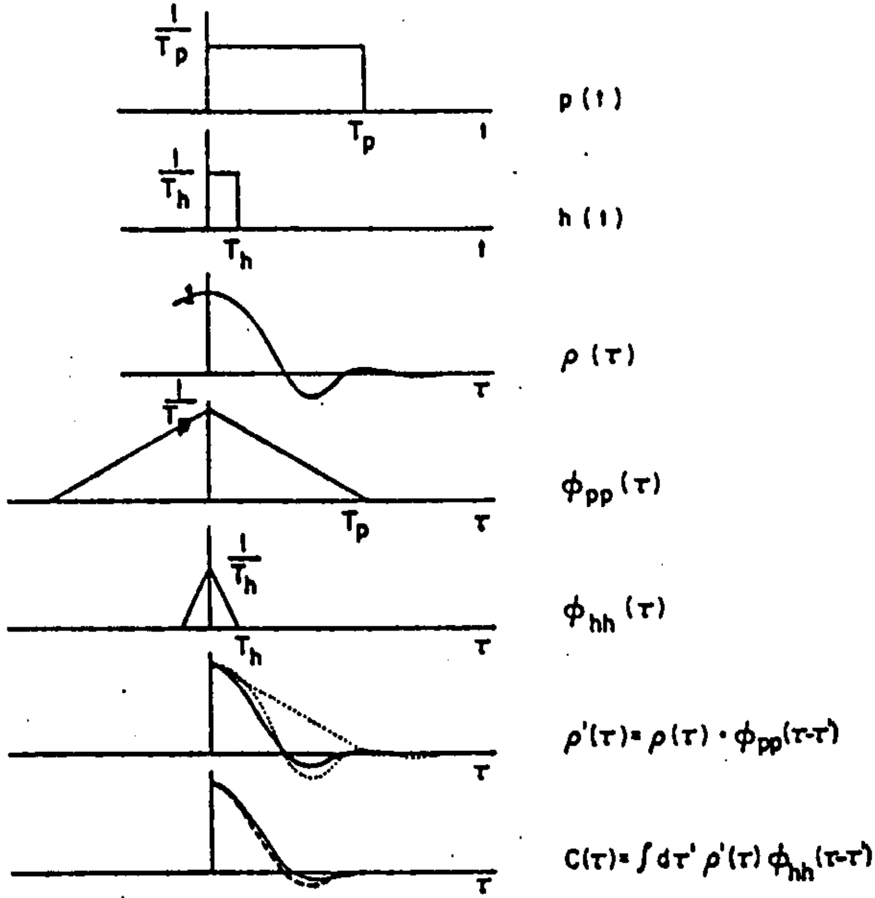
or:

$$C(\tau,t) = CK^2A(t)\int d\tau'\ \hat{\rho}_t(k,\tau')\ \phi_{pp}(\tau')\ \phi_{hh}(\tau-\tau') \qquad (17)$$

where $\phi_{pp}(\tau)$ is the autocorrelation function of the pulse shape and $\phi_{hh}(\tau)$ the autocorrelation function of the filter and decoding system.

Illustrative Examples

In order to gain a better undertanding of the significance of the formulas derived for case 2 and 3 we have constructed figure 3 and 4 respectively, corresponding to two often used pulse schemes. Case 1 does not need of a illustration since in this case the spectrum of the



1/Tp
Tp
t
p (t)
1/Th
Th
t
h (t)
ρ (τ)
τ
1/Tp
Tp
τ
φpp (τ)
1/Th
Th
τ
φhh (τ)
τ
ρ'(τ)= ρ(τ) · φpp(τ-τ')
τ
C(τ)= ∫dτ' ρ'(τ) φhh(τ-τ')

Figure 4

NOTES FOR A CHANGE IN PRESENTATION

Equation (5) can be written in a more general and elegant form. The expression for O (t) in (4) is given explicitly by

$$O(t) = \int d^3\underset{\sim}{x}dt'\, K(\underset{\sim}{x})\; p(t'-T(\underset{\sim}{x}))e^{-i\omega_o T(x)}\, n(\underset{\sim}{x},t'-T_2(\underset{\sim}{x}))h(t-t') \qquad 4a)$$

making a change of the variable $t'' = t'-T_2(x)$

$$O(t) = \int d^3\underset{\sim}{x}dt''\, K(\underset{\sim}{x})\; p(t''-T_1(\underset{\sim}{x}))e^{-iwo\ T\ (\underset{\sim}{x})}h(t-t''-T_2(x)n(\underset{\sim}{x},t'') \qquad 4b)$$

we obtain

$$O(t) = \int d^3xdt'\, \chi(t;t',\underset{\sim}{x})\; n(\underset{\sim}{x},t') \quad \text{most general linear relationship} \qquad 4c)$$

An instead of equation 5 we have.

$$C(\tau, t) = \langle O(t)O(t+\tau)\rangle = \int d^3\underset{\sim}{x}'dt''d^3\underset{\sim}{x}dt''\;\; \chi\ (t;t',\underset{\sim}{x}')\;\; \chi\ \ (t+\tau;\ t'',\underset{\sim}{x}'')$$

$$\langle n(x,t')n(x'', t'')\rangle \qquad 5a)$$

or in terms of the difference variables and τ', defined by $\underset{\sim}{x}'' = \underset{\sim}{x} + \underset{\sim}{r}$ and $t'' = t' + \tau'$

$$= \int d^3\underset{\sim}{x}'d^3\ \underset{\sim}{r}\ dt'dt'\ X\ (t;t,'\underset{\sim}{x}')\ \chi^*\ (t+\tau;t'+\tau',\ \underset{\sim}{x}'+\underset{\sim}{r})\ \rho(r,\ \tau';\underset{\sim}{x}') \qquad 6a)$$

This expression is very general in scope, is relatively simple, and has a very simple interpretation. It involves only two functions $\chi\ (t;t'\underset{\sim}{x}')$ and $\rho(\ )$. The first is a characteristic of the system and can be interpreted as the instrument response as a function of time, t, to the instantaneous presence of a single scatterer at point x' in space at time t'. It includes the effects of antennas, propagation, receiver, coding, decoding and filtering. The second is the space-time density (of scatterers) auto correlation function and charactherizes the fluc--tuations in the medium.

The expression can be used as the starting point in determining the functional relationship for a particular instrument.

For example in the case of spatially homogeneous and time stationary medium illuminated by a planar wave of constant amplitude and frequency and where the receiver has a very large bandwidth.

$$C(\tau) = KV^2\rho(k,\tau)$$

where V is the volume defined by the interaction of both antennas.