Inclination of the Geomagnetic Field Measured by an Incoherent Scatter Technique

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A technique to measure the direction of the geomagnetic field by using incoherent scatter techniques is described. The technique has been used to measure the inclination of the magnetic field at the Jicamarca Radar Observatory with an accuracy of the order of 1 minute of arc in the height range from 200 to 800 km. The corresponding inclinations obtained from the Goddard Space Flight Center (12/66) and the International Geomagnetic Reference Field (10/68) models are found to be about 1° in error.

The incoherent scatter technique for ionospheric research has more than satisfied the expectations of those who originally proposed it [Gordon, 1958; Bowles, 1958] and has proven its ability to measure most of the parameters of interest that define the ionosphere. Measurements of electron density, ionic composition, electron temperature, ion temperature, ionospheric drifts, and electrical fields have been reported (see Evans [1969] for a review paper and references). The present paper is a description of a technique and a report of the results for measuring one additional parameter: the direction of the magnetic field at ionospheric heights.

Incoherent scatter techniques for measuring the direction of the magnetic field have already been reported [Millman, 1965; Cohen, 1970] but not with the accuracy of the technique described here.

An accurate knowledge of the direction of the magnetic field is important for the interpretation of the incoherent scatter signals. This is especially important at the Jicamarca Radar Observatory, since the autocorrelation (or the spectrum) of the signals becomes more sensitive to the direction of the magnetic field as the propagation vector of the probing wave approaches orthogonality with the magnetic field. An accurate knowledge of this direction is also important in the determination of electron densities by using the Faraday rotation angle of the backscatter echoes. The measurements reported here were motivated by this application, since it was shown that current magnetic models were not accurate enough [Cohen, 1970].

The use of this technique is not limited to the application that motivated it. The results are sufficiently accurate to make a significant contribution to the development of better models of the earth's magnetic field. It should also be possible to measure changes in field inclination in the ionosphere during magnetic storms.

**Technique**

The technique makes use of the fact that the spectrum of the scattered signal narrows when the 'observed wave vector,' \( \mathbf{k} \), is nearly perpendicular to the magnetic field. In an incoherent scatter experiment the observed wave is an electron density fluctuation wave with wave vector \( \mathbf{k} \) equal to the difference between the wave vector of the transmitted wave minus the wave vector of the scattered wave in the direction of the receiving instrument. In the case of backscatter (Jicamarca included) this wave has the direction of the transmitted wave but a wavelength half as large (\( 2\mathbf{k} \) twice as large).

A narrowing of the spectrum corresponds to a widening of the corresponding autocorrelation function. The autocorrelation function of the backscatter echoes (corresponding to a given height) is shown in Figure 1, for typical ionospheric conditions and for Jicamarca's instrumental parameters. Different curves correspond to different angles \( \alpha \), where \( \alpha \) is the complement of the angle between the observed wave vector \( \mathbf{k} \) and the direction of the magnetic field. The
theory and the technique involved in the theoretical
generation of these curves have been described elsewhere [Woodman, 1967].

In Figure 1 it is important that for angles, \( \alpha \), from 90° (parallel to \( B \)) to about 2°, there is
very little change in the shape of the autocorrelation function. But, as one approaches
perpendicularity, the autocorrelation function widens very rapidly until, when \( \alpha = 0 \), it
becomes a constant (if one assumes a collisionless plasma) regardless of time delay, \( \tau \).
The autocorrelation function for a large enough \( r \) and for different values of \( \alpha \) is a sharp function of \( \alpha \).
Figure 2 shows this dependence on \( \alpha \) for a time delay of 6.67 m/sec (a delay used in the experiment).
Notice that at this delay the region of the ionosphere that contributes correlated
echoes is confined to a fraction of a tenth of a degree centered around perpendicularity to the
magnetic field. The object here is to locate this region for each height, i.e., to find the locus
where \( k \) is perpendicular to the magnetic field. At Jicamarca (2° dipole latitude), this locus
on a vertical plane does not deviate much from the observatory's vertical. With simple geometry,
and by using model curvatures for the magnetic field lines, the inclination of the magnetic field
lines along the vertical can be derived from this locus of perpendicularity.

In what follows, we shall ignore the finite extent of Jicamarca's antenna beamwidth (less
than 1°) in the magnetic east-west direction. Magnetic field properties are a very slow function
of magnetic longitude, and there is almost no change within the east-west width of the
antenna beam. This allows us to reduce our geometry to a two-dimensional one in the plane
defined by the magnetic north-south axis of the antenna and the center axis of the antenna beam.

The axis of the antenna beam is not exactly vertical but has a tilt of 1.50° to the SW. We
have made corrections to take this tilt into account, but for the sake of clarity we shall
ignore the small tilt to the west in our presentation.

A technique used in radio astronomy to determine the position of point radio noise sources
(radio stars) is cross-correlation interferometry. With this technique, the random signals received
at two spaced antennas are cross-correlated. The cross-correlation can be represented by a complex
number whose amplitude depends on the size of the source and the phase on the source
position with respect to the center plane perpendicular to the interferometer axis. The
 technique used to measure the direction of the magnetic field is an extension of this technique;
the main difference is that the signal of one antenna is delayed by a relatively long time
(of the order of 6 msec) before it is correlated with the other.

As we have seen, backscatter signals that correlate at long delays come from a region very
close to where \( B \) is perpendicular to \( k \). It can be shown (see the appendix) that the phase angle
of the cross-correlation depends on the angular position, \( \theta \), of this region with respect to the
interferometer center plane (see Figure 3) and also on the velocity, \( v \), of the medium along \( k \).
be more specific, if \( N(t) \) is the signal received from range \( h \) at antenna \( N \) and \( S(t) \) is the corresponding signal at antenna \( S \), then the cross correlation

\[
C_{NS}(\tau) = \langle N(t)S(t + \tau) \rangle = KC(\tau) \exp\left(-jk\nu \tau - j2\pi m\theta\right)
\]

(see the appendix) (1)

where \( KC(\tau) \) is a real function of \( \tau \) that depends on the antenna gain, the magnetic field direction, and the characteristics of the plasma; and \( m \) is the separation in wavelengths between the two antennas of the interferometer. Here \( C_{NS}, N, S, C, \) and \( \theta \) depend parametrically on the range \( h \), but we shall not show the dependence explicitly to simplify the notation. Since \( KC(\tau) \) is real, the only complex phase angle is the one given by the exponent in (1).

By cross-correlating \( S \) with \( N \) delayed we get

\[
C_{SN}(\tau) = \langle S(t)N(t + \tau) \rangle = KC(\tau) \exp\left(-jk\nu \tau + j2\pi m\theta\right)
\]

(2)

If we call the exponents in (1) and (2) \( \phi_{NS} \) and \( \phi_{SN} \) respectively, we find an expression for \( \theta \)

\[
\theta = \frac{\phi_{NS} - \phi_{SN}}{4\pi m}
\]

(3)

which gives us the angle \( \theta \) (for a given range \( h \)) at which the line of sight from the interferometer is perpendicular to \( B \). We also obtain as a byproduct, the velocity of the medium \( v \) at that particular range, from

\[
v = \frac{\phi_{NS} + \phi_{SN}}{2k\tau}
\]

(4)

This velocity could be of special interest when measuring \( \theta \) during magnetic storms.

**EXPERIMENTAL ARRANGEMENT**

The above technique was implemented at Jicamarca as follows (Figure 4). The antenna was divided into four equal square sections. The full antenna at Jicamarca is a square array of 9216 (96 x 96) crossed half-wave dipoles. The diagonals of the square are oriented in the north-south and east-west magnetic directions (9°49' and 95°49' azimuth). The east and west quarters of the antenna were excited in parallel by the transmitter. The north and south quarters were used as the two antenna elements of a cross-correlation interferometer. Pulses were sent every 6.67 m/sec (1000 km maximum range). Echoes from every other pulse were received by either the north or the south antenna, and samples of the echoes corresponding to different heights were taken. The samples \( N(h, t) \), taken from the north antenna, were stored in a digital memory and multiplied by the corresponding samples \( S(h, t + 6.67 \times 10^{-4}) \),

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**Fig. 2. Angular dependence of \( \rho(\alpha, \tau) \) for a delay, \( \tau \), of 6.67 m/sec.**

**Fig. 3. Geometrical definition of angles used in text.**
taken from the south antenna 6.67 m/sec later. A digital correlator performed the cross multiplication of each pair of samples corresponding to each sampled height, and one product was obtained every two pulses. A statistical estimate of the cross-correlation \( \langle N(h, t) S(h, t + \tau) \rangle \) for \( \tau = 6.67 \) m/sec was obtained from an average of 10\(^5\) of these products (22 min of integration).

To generate the cross-correlation \( \langle S(h, t) N(h, t + T) \rangle \), the south samples taken after the second pulse of any pair were also stored in memory, to be multiplied by the north samples of the first pulse of the following pair. Since the pulses are evenly spaced, the north samples in this instance are delayed by the same 6.67 m/sec with respect to the south ones. The accumulated correlation functions were then processed for \( \theta \) and \( v \) and plotted on line.

The technique is independent of the polarization used. We used a linear polarization on transmission. We received on both orthogonal linear polarizations (45° with respect to magnetic north) to receive the total of the backscatter power and cross power. This was necessary since the polarization used is not a normal mode of propagation through the medium and only part of the backscatter power was received on each polarization. The phase of the signal received was found to be independent of polarization, as expected.

**ON THE ACCURACY OF THE TECHNIQUE**

Figure 5 shows the results obtained by the system with 10\(^5\) samples. The error bars represent the statistical uncertainty. If \( \psi \) is the estimate of the phase angle of a complex cross-correlation function, it can be shown that [Woodman and Hagfors, 1969]

\[
\langle \psi^5 \rangle = \frac{(1 - S^\circ)^2}{2NS^\circ}
\]

where \( S \) is the ratio of the cross-correlation amplitude to the total power (signal + noise) received at any of the antennas. The experimental values obtained for \( S \) are of the order of 0.05, which gives a \( \langle \Delta \psi^5 \rangle \) of 0.045 radians for 10\(^5\) samples. The corresponding uncertainties for \( \theta \) are obtained from (3). The antenna separation \( m \) is 24.5 wavelengths, which gives an uncertainty of less than 1 min of arc, as shown in Figure 5. Uncertainties at higher heights are larger, since the region of perpendicularity lies outside the beam pattern of the antennas, and the electron density is smaller; consequently, there is less backscatter power.

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**Fig. 4.** Schematic diagram of experimental set-up to measure the direction of the magnetic field.

**Fig. 5.** Sample record of the interferometer output, showing statistical uncertainties. The integration time is 30 min. We call \( \theta \) the angle that the line of sight to perpendicularity makes with respect to the center axis of the interferometer. (\( \theta = -12.88 \), H.A. = 4m37sW)
Systematic errors can arise from inaccurate calibration of the interferometer and from ionospheric refraction. A dc bias in the system could also produce large errors, but these were eliminated by a sign flip technique similar to the one used in the standard incoherent scatter experiments at Jicamarca [Farley, 1969].

The absolute calibration of the interferometer baseline direction and of the antenna feedlines was obtained by using the same system of antennas and cables to track Hydra [H.A. = 9h 16m 37s, δ = 11.96°], a strong radio source at our latitude. A standard cross-correlation program without delays was used to track the star. A geometric survey of the antennas and feedlines can also be taken with confidence, because possible errors of a few centimeters are a very small fraction of a wavelength. The radio star calibration gave a declination of −12.890 for the interferometer center, which is only 0.01° off the declination of −12.88 obtained from a survey.

If we assume a horizontally stratified ionosphere, a wave with an angle β with respect to the vertical at the antennas will have, at ionospheric heights, an angle β' with respect to the vertical given by the refraction law.

\[
\frac{\sin \beta}{\sin \beta'} = n \approx (1 - X)^{1/3}
\]

where \(X\) is the square of the ratio of the plasma frequency to the wave frequency at the range in question. For typical F region peak densities and for a wave frequency of 50 MHz the refraction index \(n\) is of the order of 0.98. Measured values of \(\beta\) are of the order of 2°, which give errors of the order of 3′ of arc. These errors have been corrected by using values of electron densities taken the following day, at the same time and height, by backscattered Faraday techniques. The magnetic field direction measurements reported here were made on March 28, 1969, and the electron density profiles were made on the following day. Both days were magnetically quiet.

The refraction correction makes the assumption of a horizontally stratified ionosphere. Since the measurements were made during the day this is a fair assumption, especially at the lower F region heights.

Residual systematic errors should be less than the random error bars shown in Figure 3.

Results

Four independent measurements with 30 min of integration each were averaged together, corrected for refraction, and processed to give the results shown in Figure 6. The results are presented in terms of inclination, \(I\), of the magnetic field along Jicamarca's vertical (−11.95° lat., 76°32'20" long.).

The inclination can be derived from the angle \(\theta\) that the line to perpendicularity makes with respect to the interferometer axis. If we call \(\theta\), the angle that the vertical makes with respect to the interferometer axis, we can write with sufficient approximation that the inclination \(I\) is linearly related to the angle that the line of perpendicularity makes with respect to the vertical, \(\theta - \theta_0\) (Figure 3), since at Jicamarca this is a relatively small angle. We can then write

\[ I = c(\theta - \theta_0) \]
where $c$ is a height dependent factor that can be obtained from a simple dipole model or from a more sophisticated one. This factor has been found to be insensitive to the model used. The factor $c$ depends on the curvature of the magnetic field at the height in question, and this can be obtained with good accuracy even with the simple dipole model. The values used have been tabulated in Table 1.

We have not made any observation yet during disturbed ionospheric conditions; therefore we have not been able to observe fluctuations on the inclination that could be attributed to a magnetic storm.

**Comparison with Current Magnetic Field Models**

The inclination of the magnetic field obtained from current magnetic field models has also been plotted in Figure 6. These are the NASA GSFC 12/66 model [Cain et al., 1967] and the First International Geomagnetic Reference Field IGRF 10/68 [Cain, 1968]. Both models use satellite data. The rms deviations of the satellite data from the model was 11 $\gamma$ [Cain, 1968]. The discrepancy between both models and Jicamarca's measurements are of the order of 1°, which corresponds to errors of the order of 500 $\gamma$ in the direction orthogonal to the magnetic field. This discrepancy is much larger than the residual error of 11 $\gamma$ obtained from the satellite measurements. Since the instruments used in the satellites to derive the models measure only the absolute values of the magnetic field, it seems possible that the models derived agree well with the absolute value they accurately measure, but not with the proper direction.

**Appendix**

Here we shall semiheuristically derive the expressions used in equations 1 and 2 of the text. We can consider the signal $N(t)$ received at the north antenna and corresponding to a given range as being the resultant of the contributions $n(t, \theta) d\theta$ from different volume elements $d\theta$. Each of these contributions is weighted twice (once in transmission and once in reception) by the antenna beam pattern $F(\theta)$. We assume the same beam pattern in the $\phi$ plane for all the antenna quarters. We can then write

$$N(t) = \int F(\theta)n(\theta, t) d\theta \quad (A1)$$

If $n(t, \theta)$ is the signal received in the north antenna, the south antenna will receive the same signal, but it will be delayed by a time that depends on the angle of arrival $\theta$ and on the antenna separation in wavelengths. This time delay is equal to a phase shift of $2\pi \theta m$. We can then write

$$s(\theta, t) = n(\theta, t) \exp (-j2\pi \theta m) \quad (A2)$$

The cross-correlation $C_{ss}(t)$ is then given by

$$C_{ss}(\tau) = \langle N(t)N^*(t+\tau) \rangle = \int d\theta d\theta' F(\theta)n(\theta, t)n^*(\theta', t + \tau) \exp (j2\pi \theta' m) \quad (A3)$$

The signals $n(\theta, t)$ and $n(\theta', t + \tau)$ are statistically independent, unless $\theta'$ is within a Debye length from $\theta$. For all practical purposes it is equivalent to say that they are independent unless $\theta = \theta'$. The cross-correlation $\langle n(\theta, t)n^*(\theta', t + \tau) \rangle$ can then be written as

$$\langle n(\theta, t)n^*(\theta', t + \tau) \rangle = \delta(\theta - \theta')\langle n(\theta, t)n^*(\theta, t + \tau) \rangle \quad (A4)$$

where $\delta(\theta - \theta')$ is a delta function, and $\langle n(\theta, t)n^*(\theta, t + \tau) \rangle$ is just the autocorrelation of the signal $n(\theta, t)$ received at a single antenna from a direction defined by the angle $\theta$. We can write $\langle n(\theta, t)n^*(\theta, t + \tau) \rangle$ in terms of a conveniently normalized function $\rho'(\theta, \tau)$, namely

$$\langle n(\theta, t)n(\theta, t + \tau) \rangle = K\rho' (\theta, \tau) \quad (A5)$$

<table>
<thead>
<tr>
<th>$h$, km</th>
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<th>$h$, km</th>
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<tr>
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<td>200</td>
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<td>600</td>
<td>0.745</td>
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<td>0.825</td>
<td>800</td>
<td>0.670</td>
</tr>
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The problem of finding $\rho'(\theta, \tau)$ is the standard incoherent scatter problem. It has been solved by many authors (see Evans [1969] for references) considering a nonmoving (no macroscopic velocities) plasma. Woodman [1967] includes numerical values and a discussion for the case of angles close to the magnetic field; the same computer program he used has been used to generate the curves shown in Figure 1. If $\rho(\theta, \tau)$ is the correlation function for the nonmoving case, a simple change to a moving set of coordinates gives the correlation function for a moving plasma with velocity $v$ along $k$. Here,

$$\rho'(\theta, \tau) = \exp(-ijkv\tau)\rho(\theta, \tau) \quad (A6)$$

where $\exp(-ijkv\tau)$ is the only complex function. This relation is discussed in more detail, including the effects of receiver bandwidth and pulse characteristics, by Woodman and Hagfors [1969].

From (3), (4), (5), and (6) we get

$$C_{NH}(\tau) = K \exp(-jkw\tau)$$

$$\int F(\theta) \exp(j2\pi\theta/m)\rho(\theta, \tau)\, d\theta \quad (A7)$$

For large values of $\tau$, we find that $\rho(\theta, \tau)$ is a very sharp function of angle with a maximum at $\alpha = 0$ (Figure 2). Here $\alpha$ is the complement of the angle between $k$ and $B$. In terms of $\theta$, we can write $\alpha = \theta - \theta_0$, where $\theta_0$ is the angle, with respect to the interferometer axis, at which a line of sight from the receiving antenna is perpendicular to the magnetic field. Thus $\rho(\theta, \tau)$ is a sharp function of $\theta$, centered around $\theta = \theta_0$. We can then approximate (7) by

$$C_{NH}(\tau) = K \exp(-jkw\tau)$$

$$+ j2\pi\theta_0 m F(\theta_0) \int \rho(\theta, \tau)\, d\theta \quad (A8)$$

or

$$C_{NH}(\tau) = KC(\tau, \theta_0) \exp(-jkw\tau + j2\pi\theta_0m)$$

where $KC(\tau, \theta_0)$ is a real function. From the stationarity of the process we can write

$$\langle N(t)S^*(t + \tau) \rangle = \langle S^*(t)N(t - \tau) \rangle$$

$$= \left[ \langle (S(t)N^*(t - \tau)) \right]$$

therefore

$$C_{NH}(\tau) = \langle S(t)N^*(t + \tau) \rangle = C_{NH^*}(-\tau) \quad (A10)$$

$$C_{SN}(\tau) = KC(\tau, \theta_0) \exp(-jkw\tau - j\pi\theta_0m) \quad (A11)$$

We have made use of (8) and (11) in the text where we have dropped the subscript in $\theta$.

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