Radar imaging comparison methods

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Abstract

In this work, we are conducting a comparison of different methods to solve a one-dimensional aperture-synthesis radar imaging problem based on simulations. For this purpose, we are going to consider the geometry of the Jicamarca ionospheric radar. These methods are going to be applied to the generation of images of field-aligned plasma irregularities in the equatorial ionosphere, particularly, to the case of Spread-F phenomena. The methods used in the comparison goes from a direct Fourier inversion and a simple numerical integration, to more elaborated algorithms, such as, Capon's method and Maximum entropy method. We are also going to include in the comparison, the compressed sensing technique using the Haar and dab4 basis, in this case, we are assuming that the brightness function of the spread-F echoes has a sparse representation. In the simulations of the radar measurements, we are considering Gaussian shape brightness functions. The different methods will be compared based on some metrics of the reconstructed images.

Inverse problem

Data underlying radar imaging are cross correlation measurements obtained from ground-position receivers. For acquire images of field-aligned irregularities at the magnetic equator, we just need a one-dimensional array of receivers throw equatorial axis [Hysell & Chau,2006]. A schematic picture of these receivers is displayed in fig. 1.

Each receiver pair can sample visibility function V(·) in kd, where k is the radar wave number (module of vector k in fig 1) and d the relative distance between receivers ($d_j - d_i$ in fig 1). We are interested in calculate effective brightness function B(·), which represents the angular distribution of received signals. The equation that relates B(·) and V(·) is given in [Thompson,1986]:

$$V(kd) = \int_{\mathbb{R}} B(\theta) e^{ikd\sin(\theta)} d\theta \qquad (1)$$

Where θ is the zenith angle in fig. 1. We want to solve the problem for B(·). The main problem for solving (1) is that we can only get, at most, one sample of V for each pair of ground-position receivers, so, because of the reduced number of receivers in Jicamarca, the usual results on reconstruction from sampling are not useful.

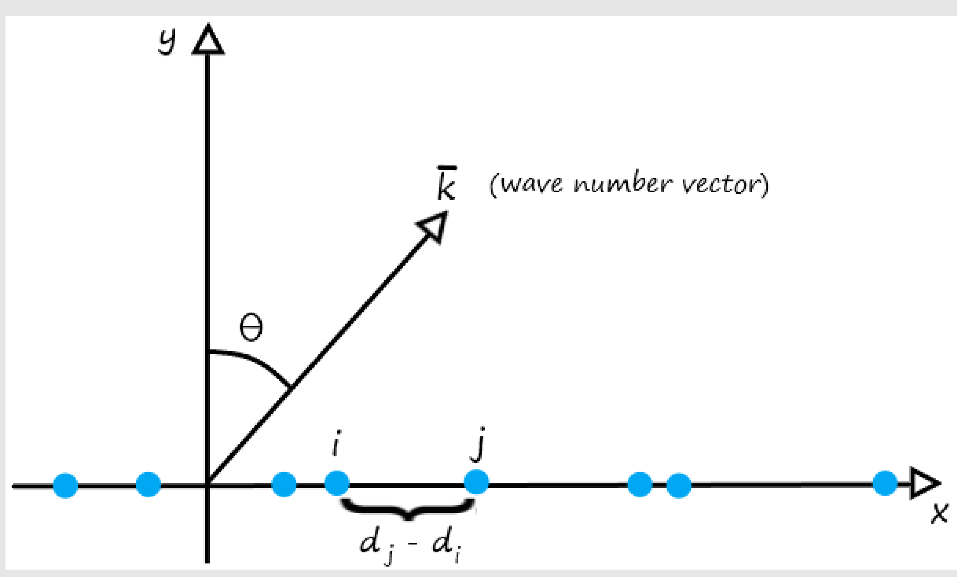


Fig. 1

Radar imaging algorithms

<u>**Fourier inverse method:**</u>
This method was introduced in [Kudeki & Sürücü,1991] and consists in approximate B by $\mathcal{F}\{V(kd)\}$, using sinθ ≈ θ in (1). Fourier transform is calculated throw the discrete formula:

$$f_{Fourier}(\theta) = (\cdots, e^{-ikd_j\sin\theta}, \cdots) \cdot R \cdot (\cdots, e^{ikd_j\sin\theta}, \cdots)^T$$

<u>**Simple numerical integration method:**</u>
This method consists in applying trapezoid rule for calculating $\mathcal{F}\{V(kd)\}$(θ) using the samples $V(kd_m)$.

<u>**Capon's method:**</u>
Capon's method [Palmer et.al.,1998] can be considered as the first elaborated method we work with. This method proposes to take as brightness function the solution of:

$$\min_{\omega(\theta)}\{\omega^\dagger(\theta) \cdot R \cdot \omega(\theta) : e^\dagger \cdot \omega(\theta) = 1\} \quad \text{where} \quad e := (e^{ikd_1\sin\theta}, \cdots, e^{ikd_8\sin\theta})$$

<u>**Maximum entropy method:**</u>
This method has a statistic nature, it is introduced in [Hysell,1996] considering the first principle of data reduction (FPDR) established in [Ables,1974]. For this method, we first establish a resolution N for the answer vector, this number is fixed as the number of pixels such that no new details appear if we add more. Consider M is the number of independent receivers pairs (that is with a relative distance different from the others) and consider the matrix $H \in \mathbb{R}^{N\times(2M+1)}$ defined by discretization of (1) and separation of real and imaginary components. Maximum entropy method takes as brightness function the solution of:

$$\max_{f\in\mathbb{R}^N}\left\{S(f)\,|\,\forall j\in\{0,\cdots,2M\} : V_j + e_j = \sum_{i=1}^{N} f_i h_{i,j} \text{ and } \Sigma \geq \sum_{j=0}^{2M}\frac{e_j^2}{\sigma_j^2}\right\}$$

$$S(f) := -\sum_{i=1}^{N} f_i \, ln\left(\frac{f_i}{\sum_{k=1}^{N} f_k}\right)$$

$V_0 := V(0), e_0$ is its noise term, σ_j is the standard deviation of e_j, Σ is a bound for error and $\forall i : h_{i,0} := 1$.

<u>**Compressed sensing method:**</u>
Compressed sensing (CS) is a method for solving undetermined linear systems which has the advantage of being robust under noise; it was first use on radar imaging in [Harding & Milla,2013]. The central idea in compressed sensing is sparsity; a vector x is called s-sparse if $\|x\|_0 \leq s$, where $\|x\|_0$ is the number of non zero components of x. Consider a noisy linear system Ax=y+e where y is known, e is a noise term with $\|e\|_2 \leq \epsilon$ and $A \in \mathbb{R}^{p\times P}$ with p < P. Define recuperation function as:

$$\Delta(Ax) := \min_{x\in\mathbb{R}^P}\{\|x\|_1 : \|Ax - y\|_2 \leq \epsilon\}$$

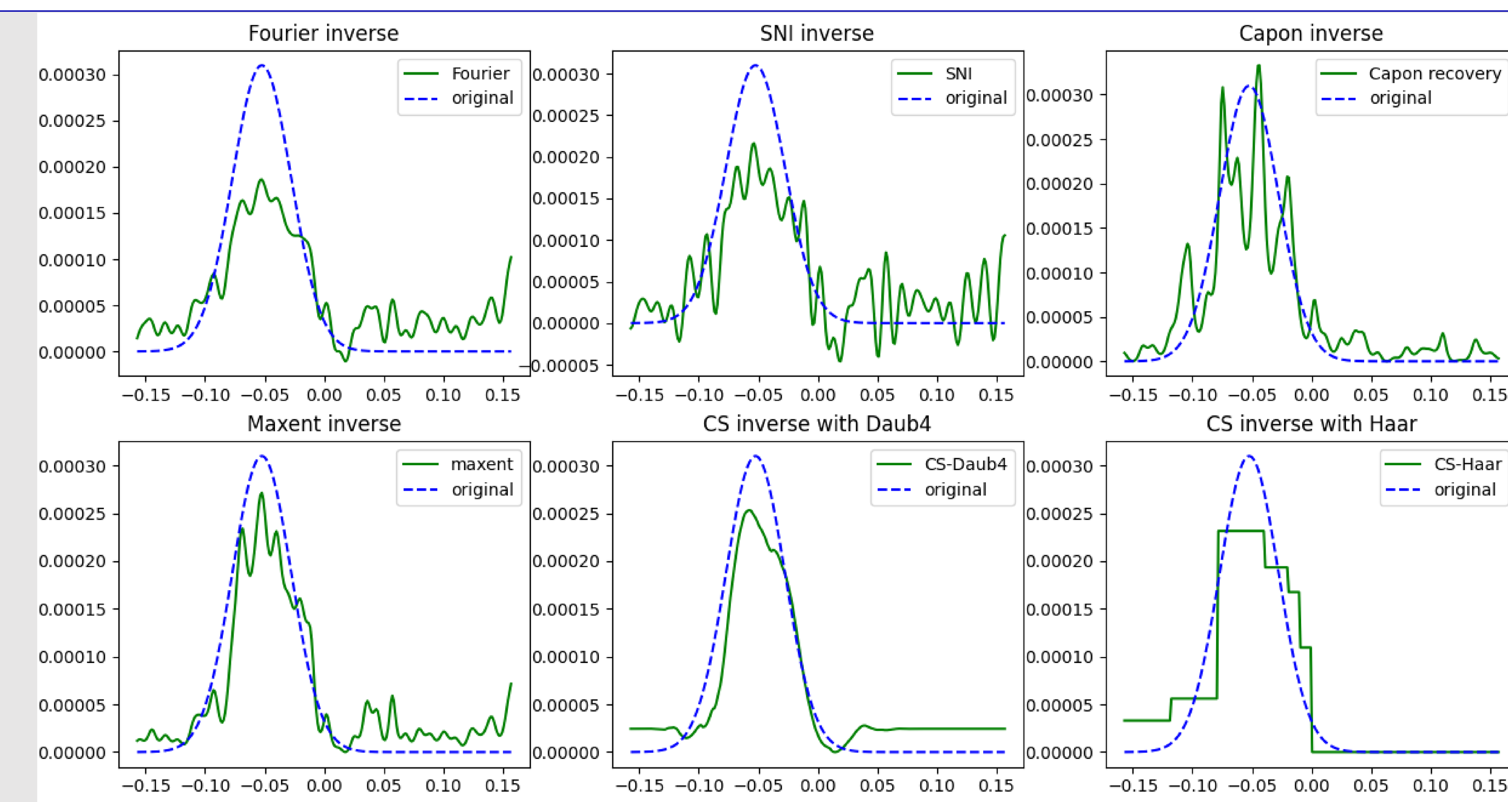
Now consider matrix H defined in maxent method, including the column $(h_{1,0}, \cdots, h_{N,0})$. Define $A = H^T$ and y=V, the discrete version of (1) is written as $Ax = y + e$. Following [Brunton,2019] and empirical arguments, we will suppose that exists a basis in which x has a sparse representation; define Ψ as the basis changing matrix and now we have the problem $A\Psi x = y + e$. Under assumption of low mutual coherence between A and Ψ (see [Harding & Milla,2013]) we can say that $\Delta(A\Psi x)$ is a good approximation of brightness function.

The choice of Ψ is an important part of CS, it determines the performance of the inversion. Some tools in harmonic analysis allow as to construct basis with desirable properties. In [Daubechies,1988], orthogonal compact support wavelets were introduced, which are a basis of L^2 with the desirable property of produce sparse representation vectors.

Simulation & results

As mentioned in abstract, we are going to simulate brightness function B by $f(\theta) := 3e^{-(\theta-\mu)^2/\sigma^2}$ where μ:=−3π/180 and σ:= 2π/180; replacing f in (1) we calculate $V(kd_{i,j})$ for each $d_{i,j}$ such that there exist a pair or receivers in Jicamarca geometry with a relative distance of $d_{i,j}$; finally, we add noise using a Gaussian distribution with mean 0 and variance 1. The resulting simulated samplings are storage on matrix R, in which the element $R_{i,j}$ is the noised value of $V(kd_{i,j})$. Note from fig. 1 that we have 8 receivers so $R \in \mathbb{C}^{8\times8}$.

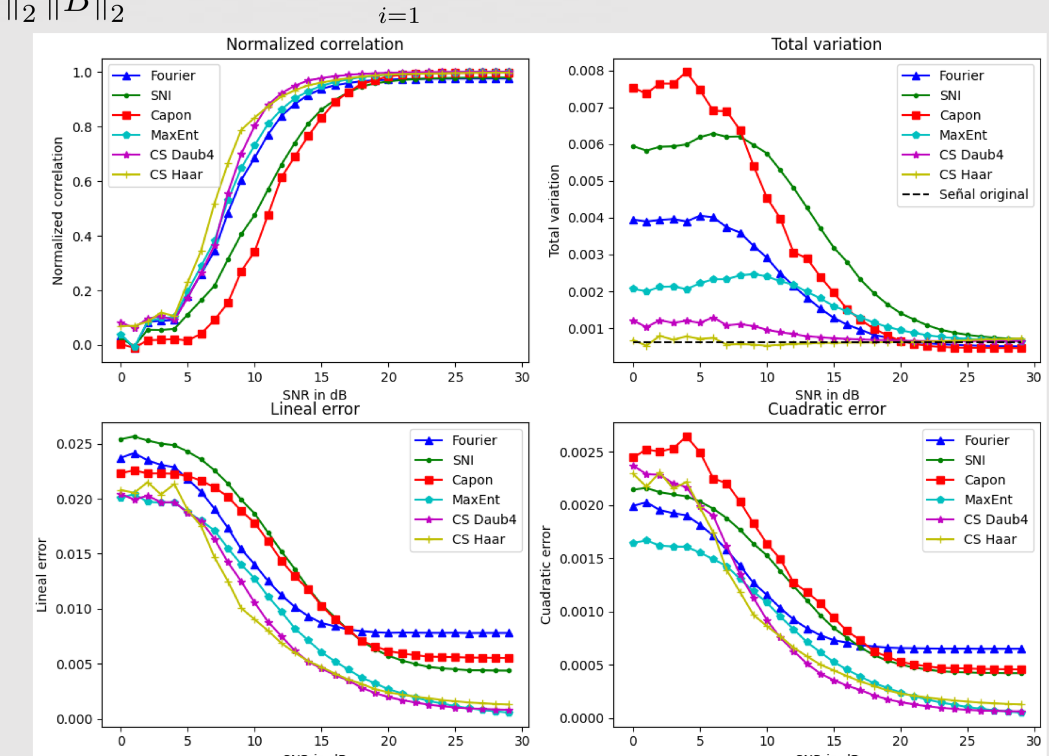
Now we display the solution of (1) using the methods described before, with a noise level of 15 dB.



Comparison

Now we compare the performance of each method by using the following metrics on recovery images: normalized correlation, total variation, linear error and squared error. These are defined by:

$$corr(f, B) := \frac{\langle f - \hat{f}, B - \hat{B}\rangle}{\|f\|_2\|B\|_2},\; TV(x, y) := \sum_{i=1}^{N-1} f[i+1] - f[i],\;\; e_1(f) := \|f - B\|_1,\;\; e_2(f) := \|f - B\|_2$$



Discussion

in [Mallat,1989], multiresolution analysis (MRA) was developed, which is a framework where wavelets can be naturally constructed. The work of Mallat opened the door for constructing personalized basis for our interest, nevertheless, it has the disadvantage of produce non compact support wavelets, which can not be put in a matrix Ψ.

Despite the disadvantage of MRA, we can go further of wavelets basis by using wavelets packets, a generalization of wavelets, introduced in [Coifmanet.al.,1990]. This construction produce a family of basis of L^2 and we can choose the best one for our objectives.

The procedure is as follows: we start with a scaling function ϕ, then produce a wavelet ψ (by using for example MRA) and define the wavelet packet $\{\mu_n\}_{n\in\mathbb{N}_0} \subset L^1 \cap L^2$ by: $\mu_0(x) := \phi(x)$, $\mu_1(x) := \psi(x)$ and inductively:

$$\mu_{2n}(x) := \sqrt{2}\sum_{k=0}^{N} h_k\mu_n(2x - k) \qquad \mu_{2n+1}(x) := \sqrt{2}\sum_{k=0}^{N} g_k\mu_n(2x - k)$$

where h_k, g_k are constants depending on ϕ. Using the notation $\mu_{n,j,k}(x) := 2^{-j/2}\mu_n(2^{-j}x - k)$ we define the spaces $U_j^n := \langle\{\mu_{n,j,k}(x)\}_{k\in\mathbb{Z}}\rangle$, which have the properties: 1) $U_j^n = U_{j+1}^{2n} \oplus U_{j+1}^{2n+1}$ and 2) there exists j_0 such that $\tilde{f} \in U_{j_0}^0$ is a good approximation of f. We can decompose $U_{j_0}^0$ using 1) in many different ways, each one corresponding with a basis and a basis changing matrix Ψ.

Conclusion

Results on comparison section show that it is not clear which method is the best one because the performance changes with the comparison metric and the noise level. Nevertheless, we can say that CS and MaxEnt work better than the other methods under almost any metric and noise level. This can be seen specifically in metrics e1 and e2, where Fourier, SNI and Capon converge to a performance that is not as good as the one where CS and MaxEnt converge. Our results also show that CS with dab4 wavelet produce a smoother result than the others, at the same time that has one of the better recovery performances.

Future work

Remains pending apply wavelets packet for Dab4 basis, and develop an optimization algorithm for optimal basis selection. Furthermore, the are some mathematical tools that allow as to construct orthonormal compact wavelets from MRA constructions, for specific cases; it would be interesting to generalize this tools and construct wavelets and wavelets packets for our specific objective.

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