# Modeling the incoherent scatter radar spectrum perpendicular to the Earth's magnetic field

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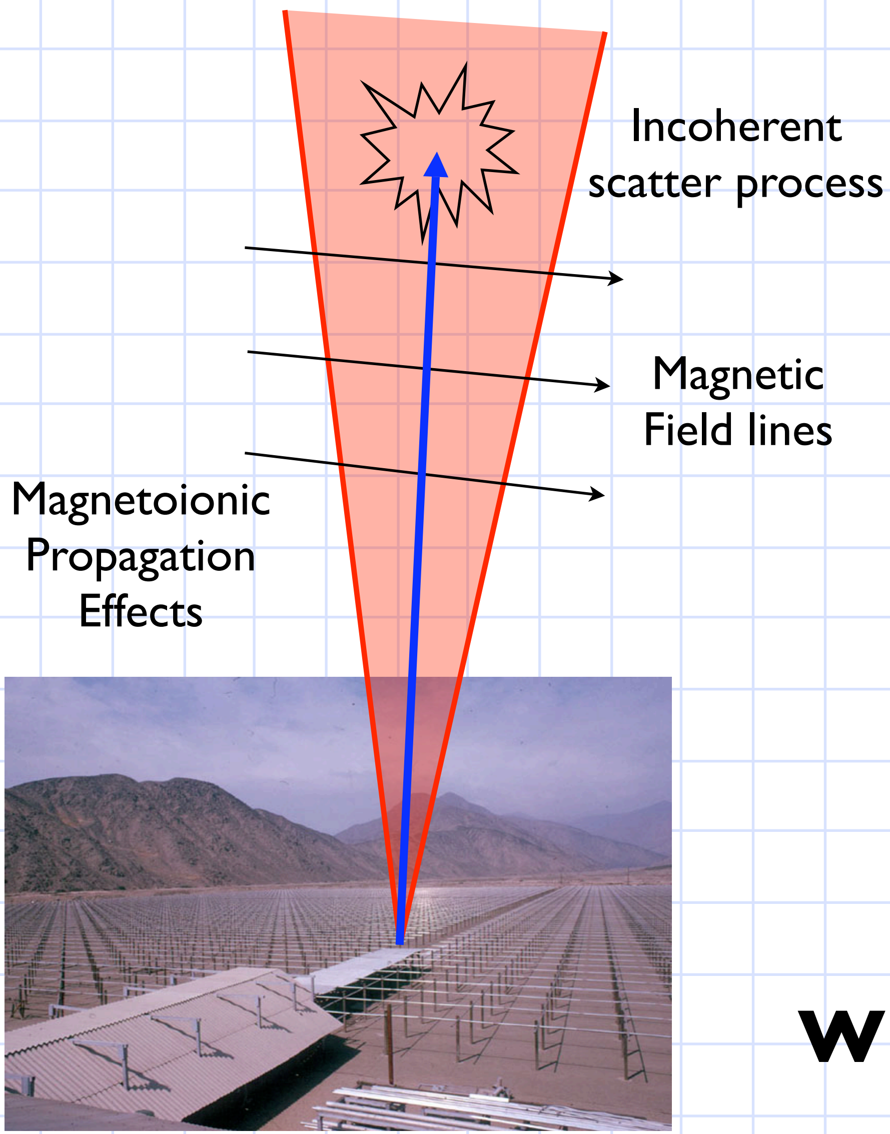
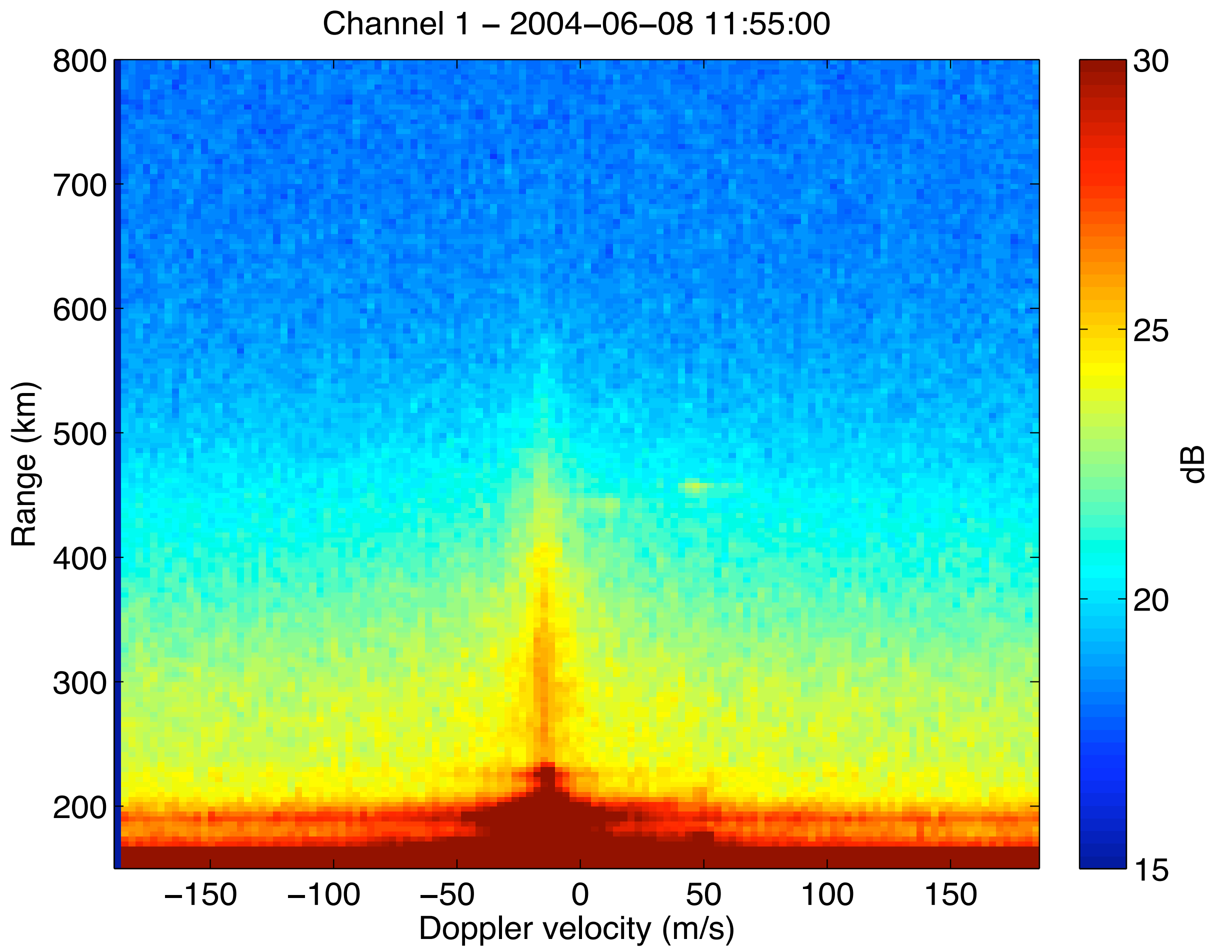
# Jicamarca ISR measurements perp. to B




- Phasing Jicamarca antenna perp. to B, we can measure:
  - Drifts:
    - Old days, using the phase of the pulse-to-pulse correlation (Woodman & Hagfors, 1969).
    - Modern times, using Kudeki et al (1999) spectral technique (Doppler shift of ISR signal).
  - Densities: using the "Differential phase" technique introduced by Kudeki et al (2003).

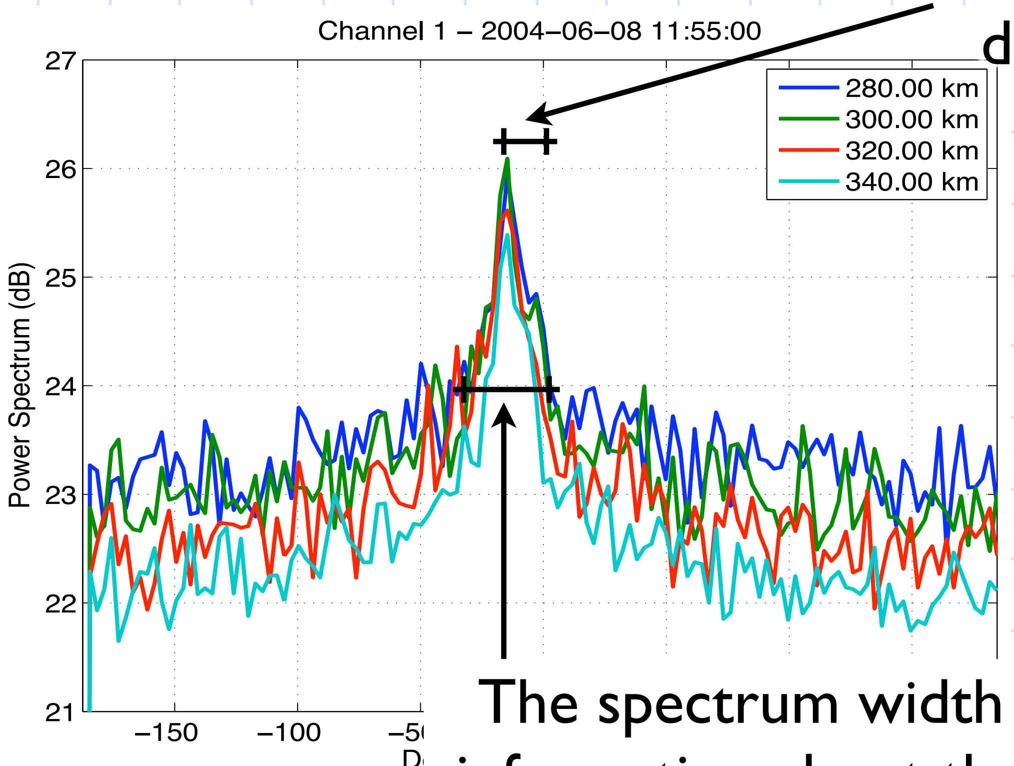
**What about temperatures?**

# Jicamarca ISR spectrum perp. to B



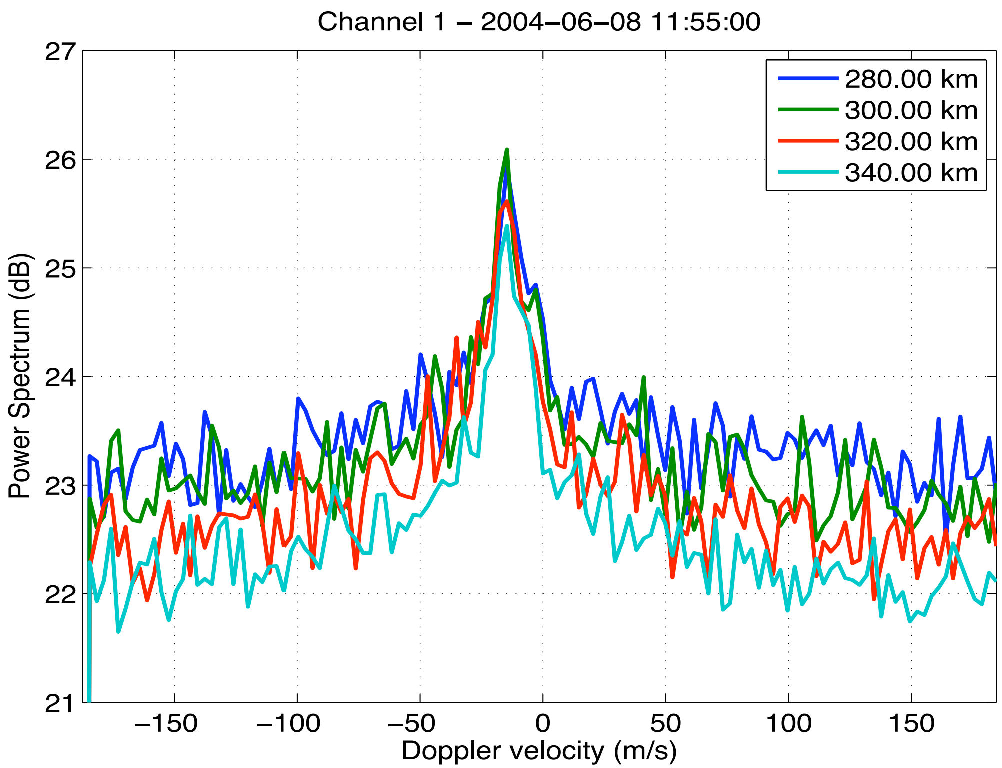
# Jicamarca ISR spectrum perp. to B

Doppler shift of the spectrum is a direct measurement of the drift.




The spectrum width should give us information about the temperatures.

# Jicamarca ISR spectrum perp. to B

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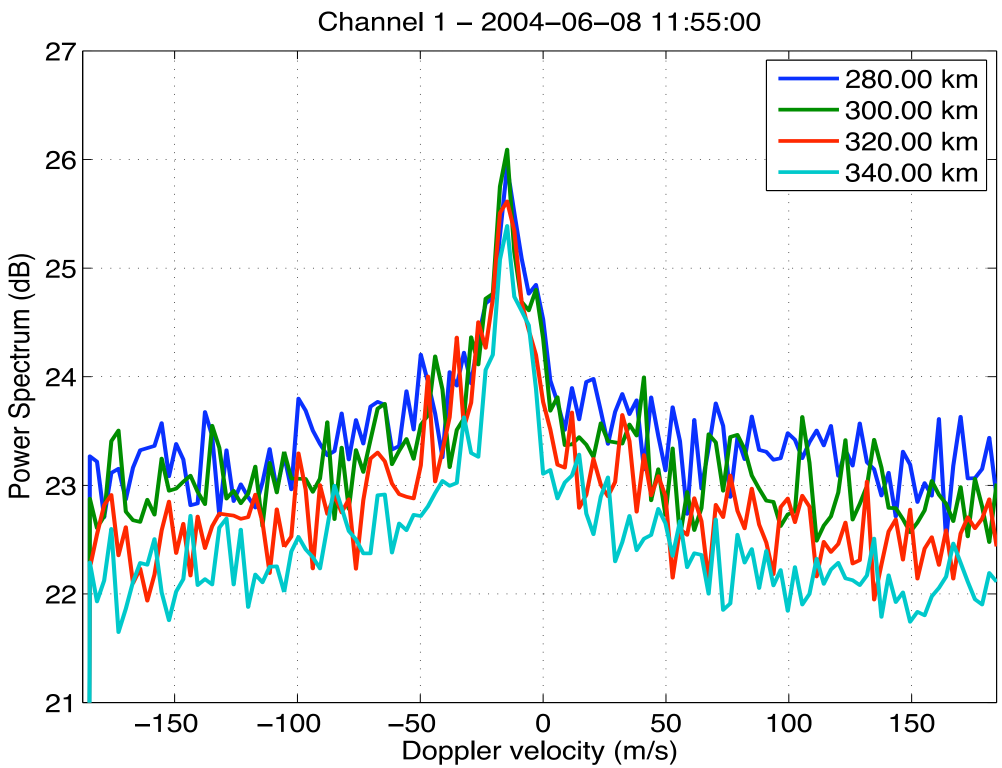
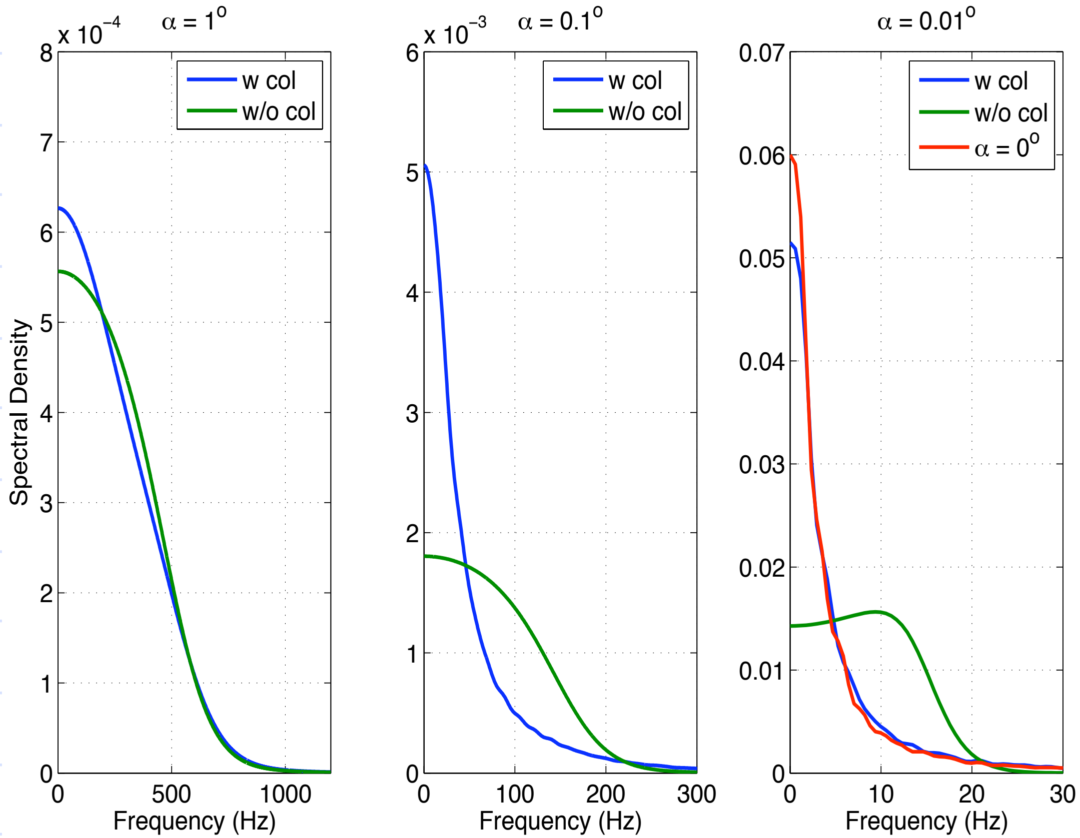
# Jicamarca ISR spectrum perp. to B




Kudeki et al (1999) analyzed the spectrum using the collisionless IS theory.
But, the temperatures they obtained were about half of what is expected.

The measured spectrum was narrower than what collisionless theory predicts.
Therefore, a revision of the IS theory was needed.

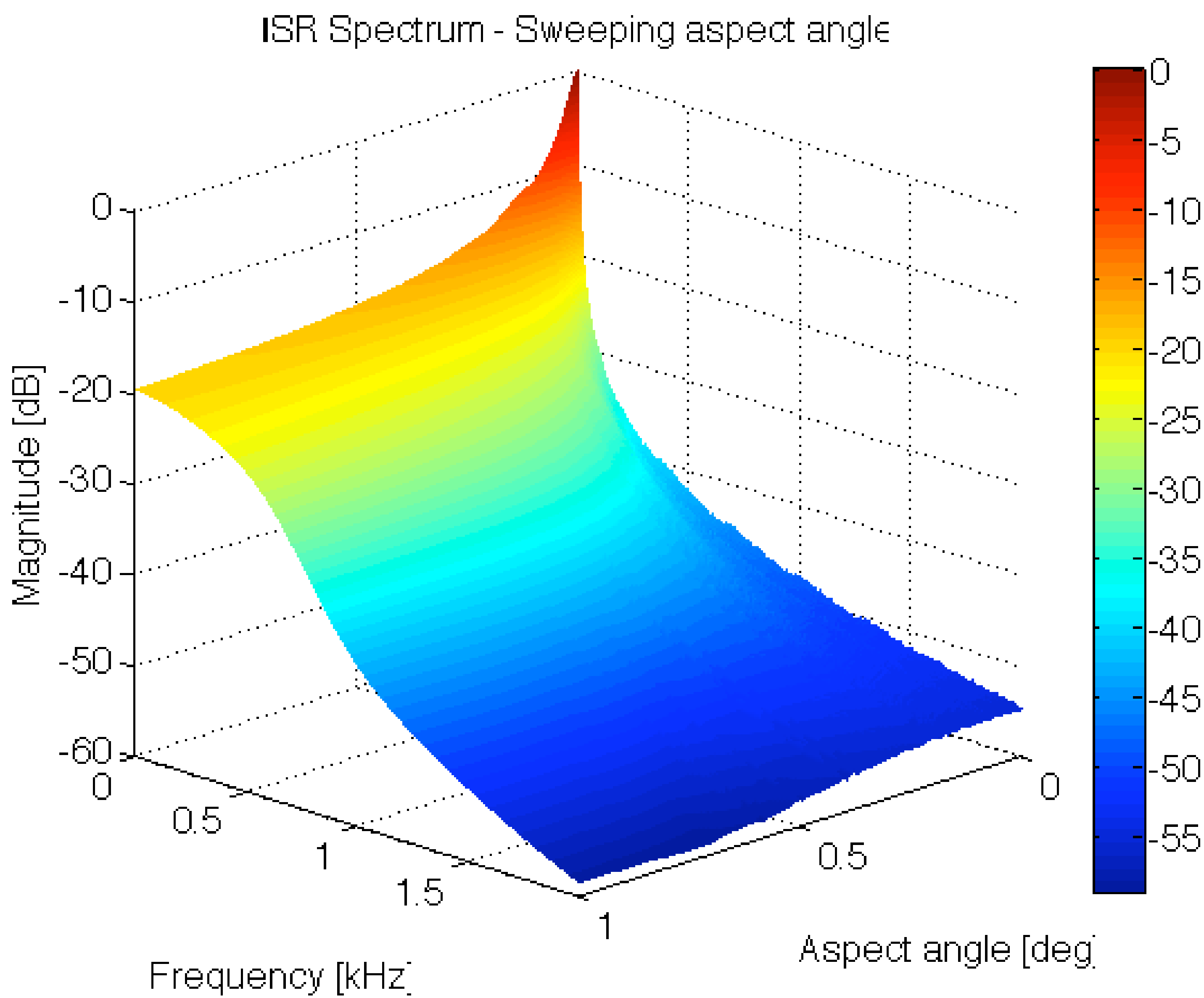
# The effect of electron Coulomb collisions




- Sulzer & González (1999) showed that Coulomb collisions are responsible for the narrowing of the ISR spectrum as the radar beam approaches perp. to B.
- Although, their IS spectrum model was developed for small magnetic aspect angles ($\alpha > 0.125°$), it was not valid for perpendicular to B.

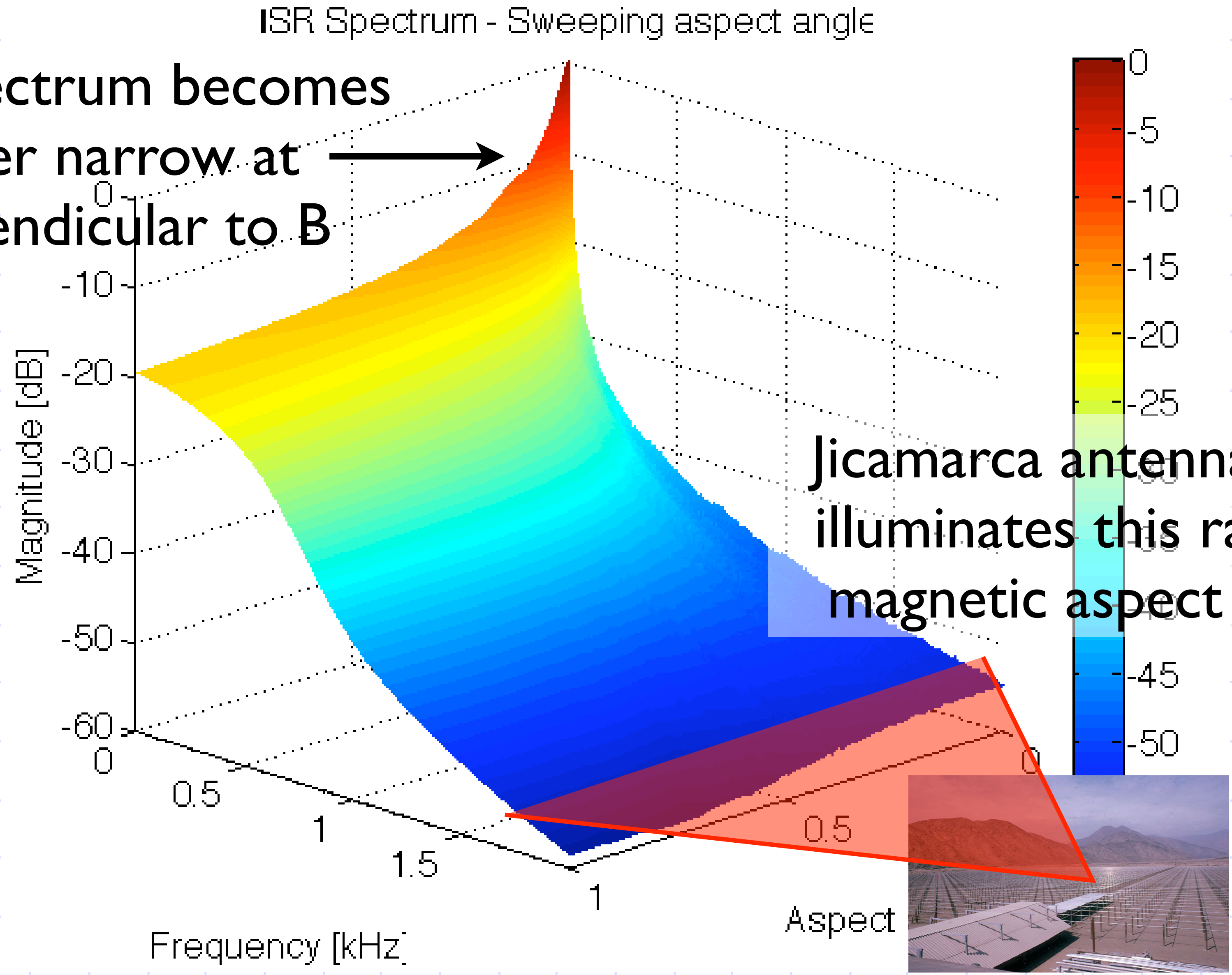
Our work started at this point. We have extended the approach of S&G and considered the effect of collisions at all aspect angles including exact perpendicularity ($\alpha = 0.0°$).

# Collisional IS Spectrum



# Collisional IS Spectrum

The spectrum becomes super narrow at perpendicular to B




Jicamarca antenna beam illuminates this range of magnetic aspect angles

# How do we include the effect of collisions?

- The standard theory of incoherent scatter formulates the spectrum of the ISR signal in terms of the so-called Gordeyev integrals

$$\frac{\langle |n_e(\omega,\vec{k})|^2\rangle}{N_e} = \frac{|j(k^2h_e^2+\mu)+\mu\theta_i J(\theta_i)|^2}{|j(k^2h_e^2+1+\mu)+\theta_e J(\theta_e)+\mu\theta_i J(\theta_i)|^2}\frac{2\mathrm{Re}\{J(\theta_e)\}}{\sqrt{2}kC_e} + \frac{|j+\theta_e J(\theta_e)|^2}{|j(k^2h_e^2+1+\mu)+\theta_e J(\theta_e)+\mu\theta_i J(\theta_i)|^2}\frac{2\mathrm{Re}\{J(\theta_i)\}}{\sqrt{2}kC_i}$$

- The Gordeyev integrals can be interpreted as the one sided Fourier transform of the correlation of the signal scattered by a singled-out test particle in a plasma where collective interactions have been neglected.

$$J_s(\omega) = \int_0^\infty d\tau e^{-j\omega\tau}\langle e^{j\vec{k}\cdot\Delta\vec{r}_s}\rangle \qquad \langle e^{j\vec{k}\cdot\Delta\vec{r}_s}\rangle = \langle e^{j\vec{k}\cdot(\vec{r}_s(t+\tau)-\vec{r}_s(t))}\rangle$$

- Thus, if the test particle trajectories were known, we could compute the single particle ACFs and corresponding Gordeyev integrals. The effect of collisions is considered in modeling the particle trajectories.

# Modeling electron and ion trajectories

- Generalized Langevin equation:

$$\begin{aligned}\frac{d\vec{v}(t)}{dt} - \frac{q}{m}\vec{v}(t) \times \vec{B} &= -\beta(v)\vec{v}(t) + \sqrt{\frac{D_\perp(v)}{2}}\Gamma_1(t)\hat{v}_{\perp 1}(t) + \\ &\sqrt{\frac{D_\perp(v)}{2}}\Gamma_2(t)\hat{v}_{\perp 2}(t) + \sqrt{D_\parallel(v)}\Gamma_3(t)\hat{v}_\parallel(t)\end{aligned}$$

- Fokker-Planck kinetic equation:

$$\frac{\partial f_s}{\partial t} + \vec{v} \cdot \nabla_r f_s + \frac{q_s}{m_s}(\vec{v} \times \vec{B}) \cdot \nabla_v f_s = \left(\frac{\delta f_s}{\delta t}\right)_{coll}$$

$$\left(\frac{\delta f_s}{\delta t}\right)_{coll} = -\nabla_v \langle \frac{\Delta \vec{v}}{\Delta t} \rangle_s f_s + \frac{1}{2}\nabla_v \nabla_v : \left[\langle \frac{\Delta \vec{v} \Delta \vec{v}}{\Delta t} \rangle_s f_s\right]$$

- These equations describe the same process as first showed by Chandrasekhar (1943). We preferred the Langevin approach because it gives us more insight into the physics of the problem.

# Friction and diffusion coefficients

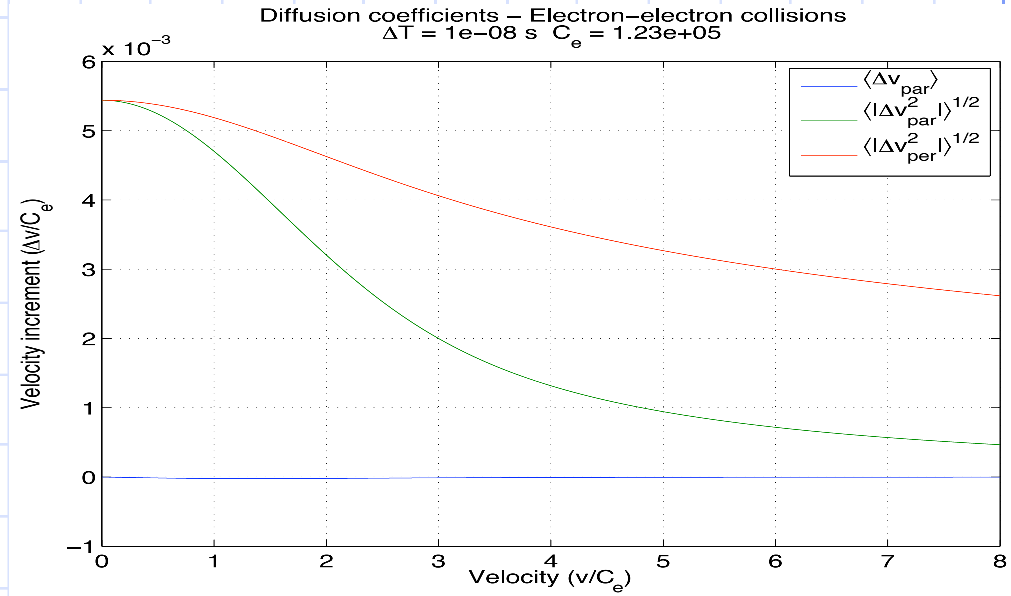
- Fokker-Planck friction and diffusion coefficients were calculated by Chandrasekhar, Rosenbluth, Spitzer and others,

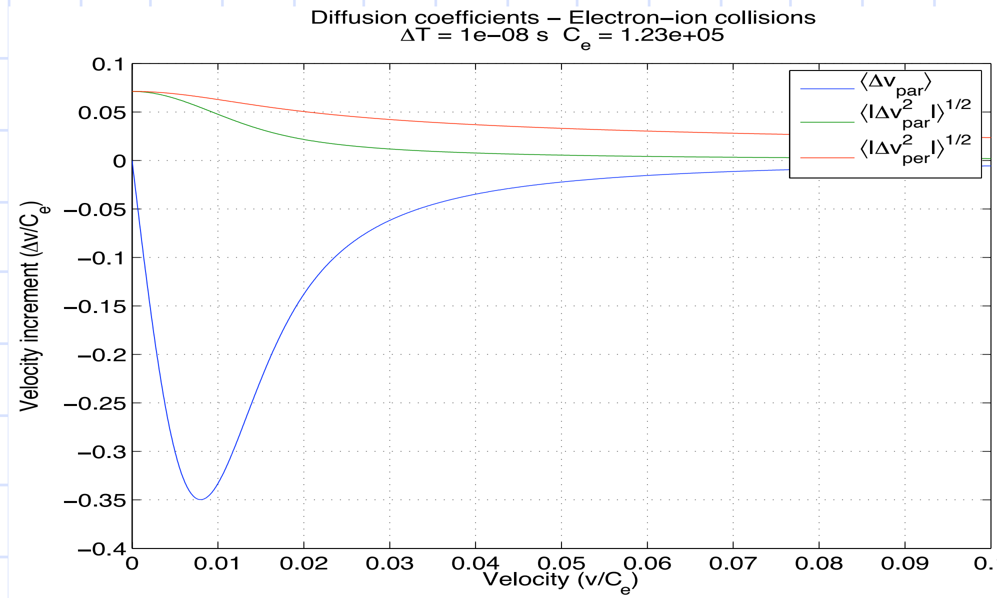
$$\langle\frac{\Delta v_{\|}}{\Delta t}\rangle = -A_D l_s^2\left(1+\frac{m_e}{m_s}\right)G(l_s v)$$

$$\langle\frac{(\Delta v_{\|})^2}{\Delta t}\rangle = \frac{A_D}{v}G(l_s v)$$

$$\langle\frac{(\Delta v_{\perp})^2}{\Delta t}\rangle = \frac{A_D}{v}\left(\phi(l_s v)-G(l_s v)\right)$$

- S&G (1999) used these coefficients in their model, however, they neglected the effect of diffusion across B.



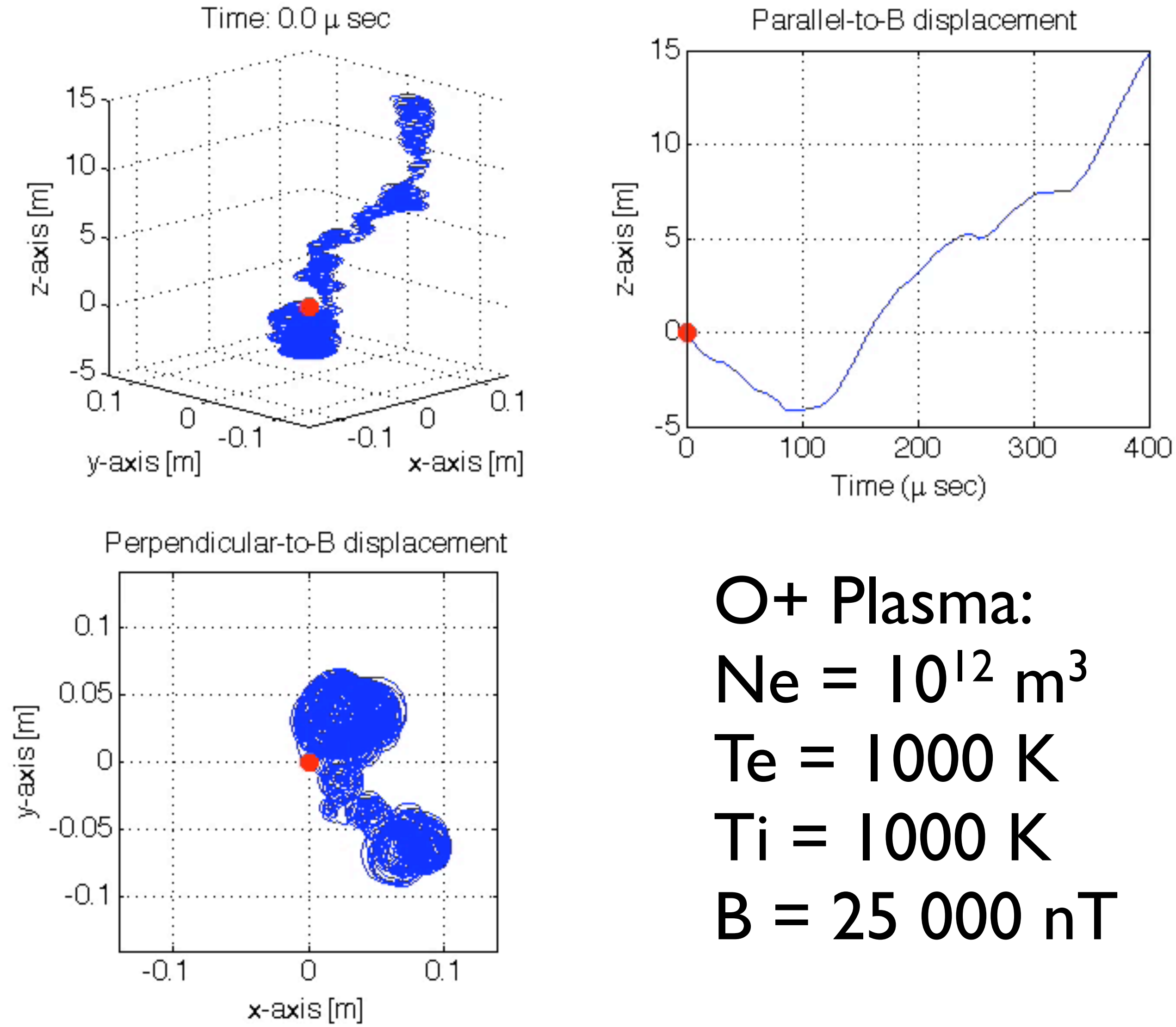
# Computer simulations

- Assumption: the magnetic field is weak enough such that within a Debye cube the trajectories of electrons and ions have a very small curvature (caused by the magnetic field). Under this condition, the Rosenbluth/ Spitzer velocity dependent coefficients are a good model for collisions.
- We simulate the motion of the test particles in the plasma using

$$\vec{v}_{i+1} = \vec{v}_i + \frac{q}{m}\vec{v}_i \times \vec{B}\Delta t - \beta(v_i)\vec{v}_i\Delta t + \sqrt{\frac{\Delta t D_\perp(v_i)}{2}}n_1\hat{v}_{i,\perp 1} + \sqrt{\frac{\Delta t D_\perp(v_i)}{2}}n_2\hat{v}_{i,\perp 2} + \sqrt{\Delta t D_\parallel(v_i)}n_3\hat{v}_{i,\parallel}$$

  for the velocities, and the trajectories are computed by numerical integration of the velocity series.
- For a given plasma configuration, the simulations run for several hours (about a day) to obtain good statistics of the particle trajectories.
- Using these trajectories, we have computed the PDFs (histograms) of the displacements and the corresponding Gordeyev integrals.

# 3D charged particle trajectories




O+ Plasma:

Ne = $10^{12}$ $m^3$

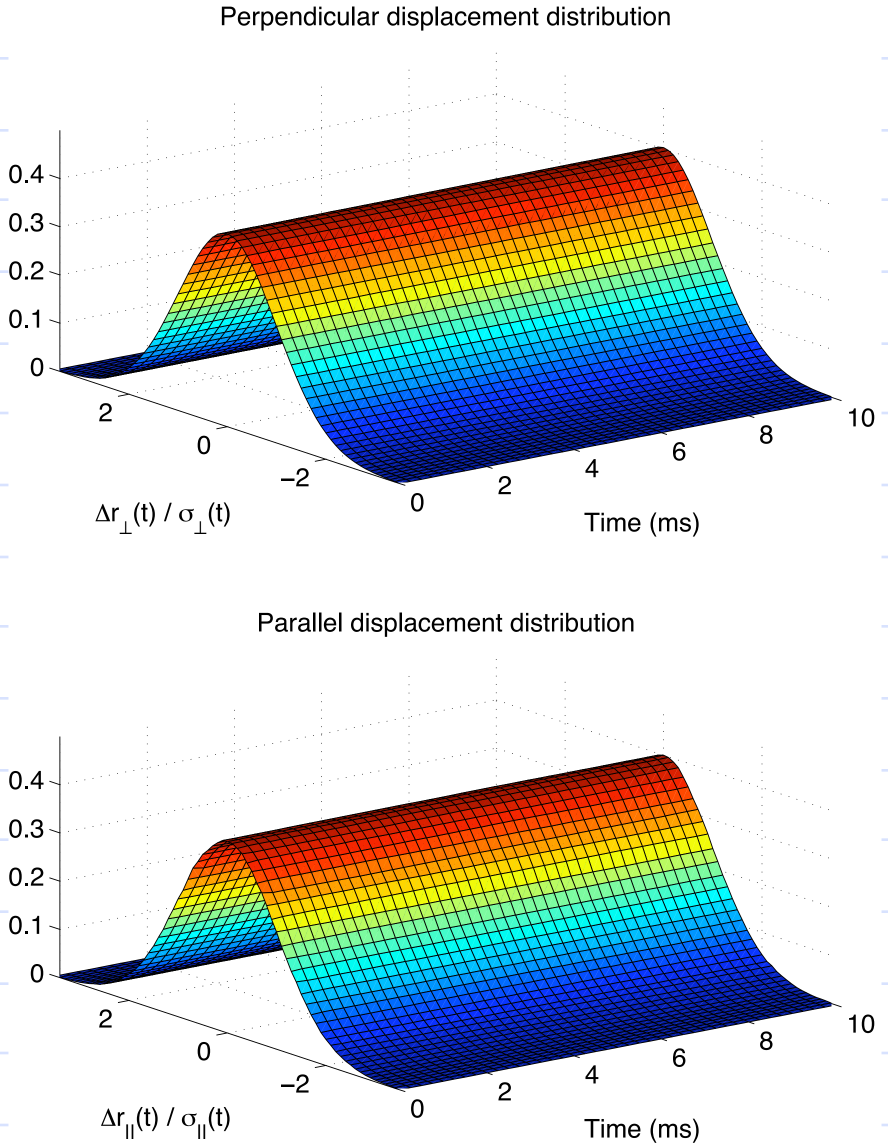
Te = 1000 K

Ti = 1000 K

B = 25 000 nT

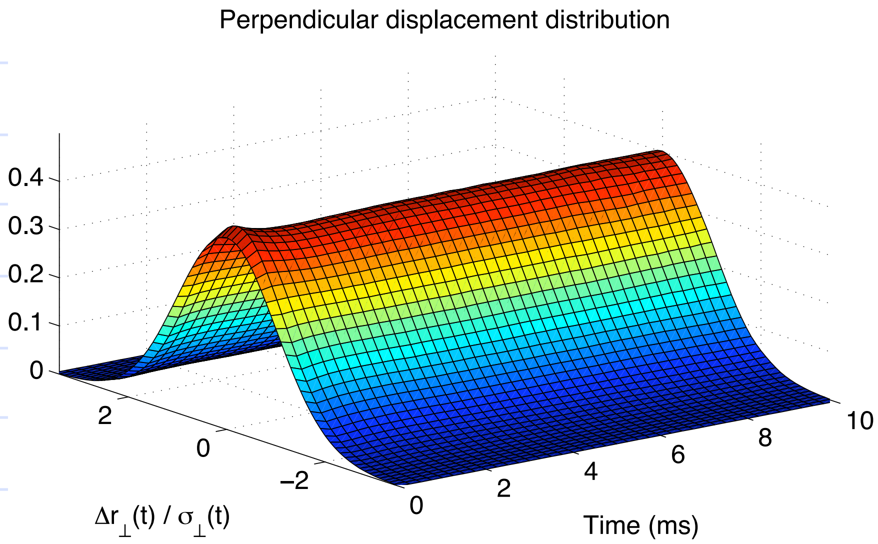
# Statistics of ion trajectories

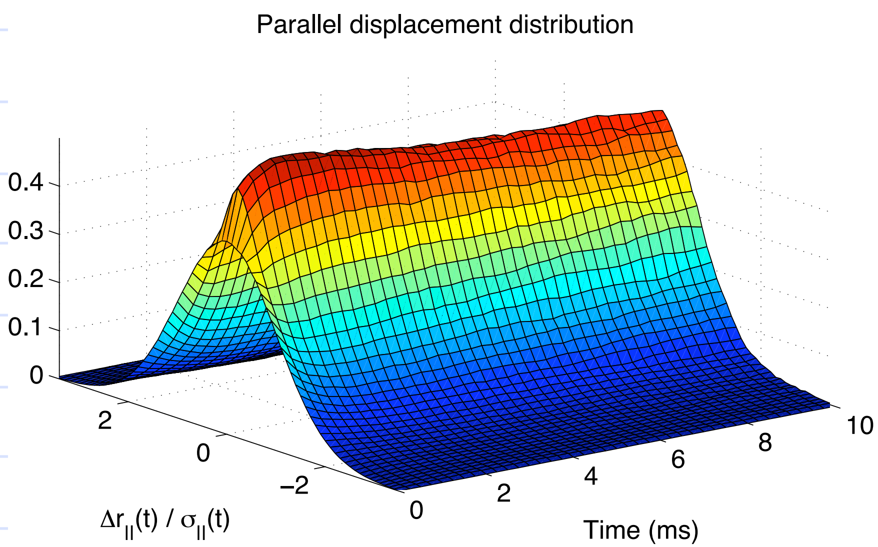
- Velocity probability distribution has a gaussian shape.
- PDF of the displacement in the direction perpendicular to B is gaussian as function of delay τ.
- In the parallel direction, the PDF also looks gaussian.
- A Brownian motion model with gaussian trajectories is a good representation of the process (Woodman, 1967).



# Statistics of electron trajectories

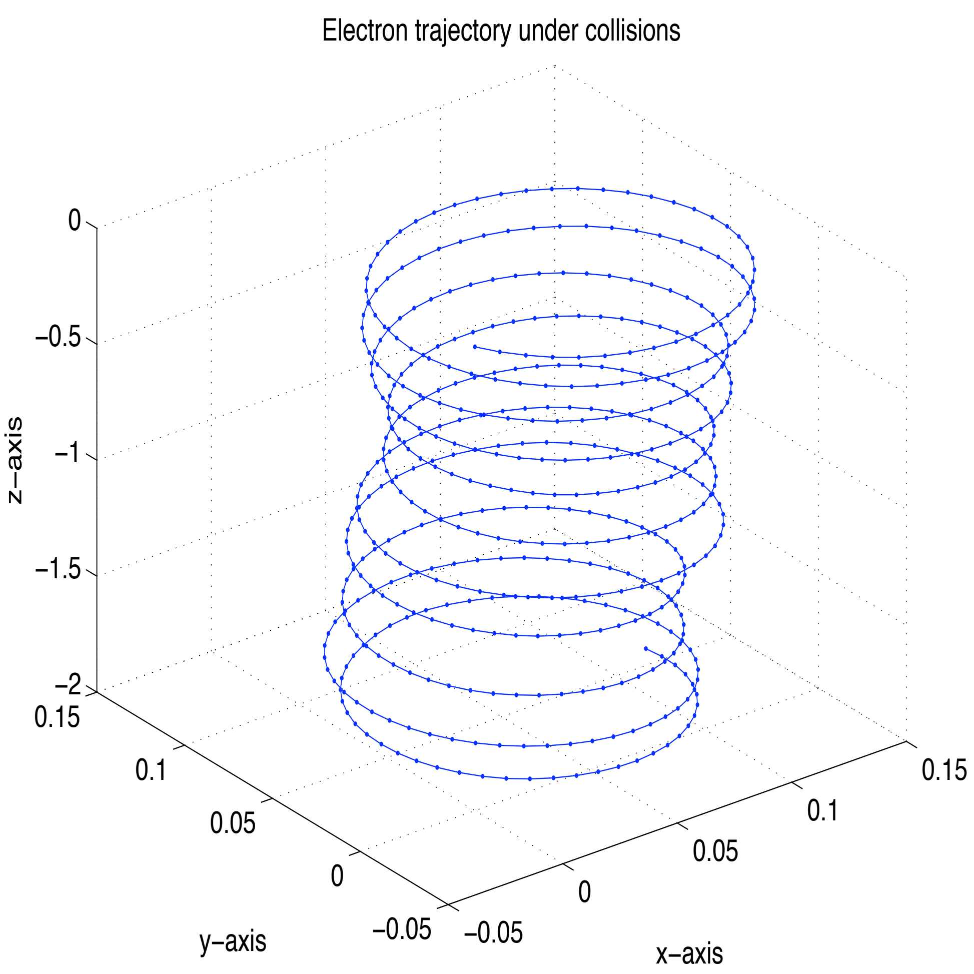
- Velocity probability distribution has also a gaussian shape.
- PDF of the displacement in the direction perp. to B is almost gaussian as function of delay τ.
- In the parallel direction, the PDF looks gaussian at short time, but rapidly becomes narrower.
- The Brownian motion model that assumes constant friction and diffusion coefficients is not a good approximation.



# Computer requirements of the simulation

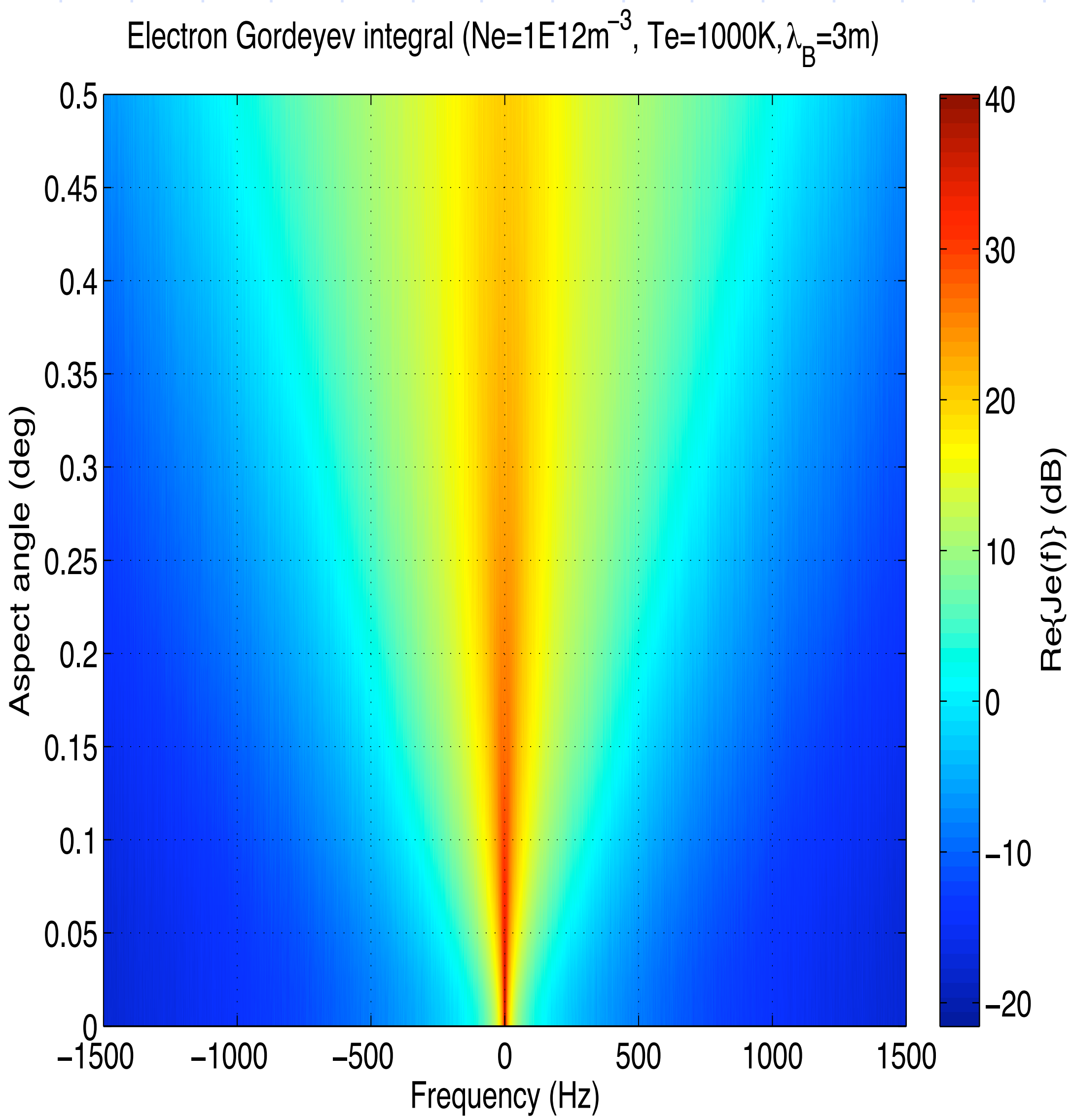
- $10^4$ sequences of $2^{17}$ samples are generated (30 GB), however, only, the statistics (Gordeyev integrals) are stored (60 MB of data).
- For a set of plasma parameters the simulation of electron trajectories takes around one day.
- For instance:
  Ne = $10^{12}$ m$^{-3}$, B = 25000 nT,
  Ti = 600:200:2000 K (8 values)
  Te = 600:200:3000 K (13 values)
  to compute the trajectories and their statistics will take about 104 days running in a single computer.



# Building the database of Gordeyev integrals

- Turing Cluster:
  768 Apple Xserves, each with two 2 GHz G5 processors and 4 GB of RAM, making a total of 1536 processors.
- Professors and students at UIUC can use the cluster for free.
- Restrictions: every user can use upto 128 processors every day (ideally). In real life, we have access to about 64 processors every day.
- Using Turing, our task takes only 2 days, instead of 104 days.

# Database of electron Gordeyev integrals




- We have built a library for an O+ plasma that considers
  - 600K < Te < 3000K
  - 600K < Ti < 2000K
  - |B| = 20, 25, 30 μT
  - Ne = 1E11, 1E12, 1E13 m$^{-3}$
  - Large set of aspect angles from 0° to 90°.

- A web-page with the results is available at http://collisions.csl.uiuc.edu/database/gordeyev/

# Does the model fit the data?

Does the model fit the data?

Please wait for the next talk...