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A renormalization group approach to estimation of anomalous diffusion in the unstable equatorial F region

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Abstract. An expression is derived for the anomalous diffusion coefficient associated with collisional interchange turbulence in the equatorial F region ionosphere. Waves with $\omega \ll \nu_{in} \ll \Omega_i$ are considered. The calculation makes use of the renormalization group method, following closely that of *Kichatinov* [1985]. The calculation is applied to the problem of plasma waves in the equatorial F region ionosphere generated by the ionospheric interchange instability. Approximations appropriate for the geometry of that problem are incorporated into the calculation. Using a model spectrum of the irregularities based on in situ satellite observations, we calculate that the anomalous diffusion seen by large-scale plasma waves can be 5 orders of magnitude larger than the ambipolar diffusion coefficient.

1. Introduction

In this paper, we investigate the nonlinear coupling that takes place between small- and large-scale waves in an unstable ionospheric plasma. It is well known that mode coupling is a means of producing a spectrum of small-scale plasma irregularities from large-scale, linearly unstable waves in the ionospheric E and F regions. Coherent scatter radar routinely make use of these irregularities to probe the ionosphere in all latitude regimes (see reviews by *Fejer and Kelley* [1980], *Riggin et al.* [1986], *Yamamoto et al.* [1991], *Haldoupis* [1989], and *Sahr and Fejer* [1996]). However, the effect of wave coupling on the large-scale waves and the background flow is not so clear. It is often neglected, the assumption being that once created, the small-scale irregularities become passive tracers of the flow.

Whereas the large-scale ionospheric waves driven by plasma instabilities are frequently highly coherent, the small-scale waves excited by nonlinear mode coupling are typically incoherent or turbulent, and the turbulence created at small scales may cause the appearance of (anomalous) dissipation. An example of such a phenomenon was described by *Ronchi et al.* [1990], who showed how small-scale gradient drift wave turbulence in the equatorial electrojet could account for the anomalously high electron mobility necessary to explain the shape of the electrojet current profile. Another example was provided by *Gary* [1980], who calculated the anomalous resistivity associated with the presence of drift wave turbulence.

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In this paper, we evaluate the anomalous dissipation in a magnetized ionospheric plasma driven by small- and intermediate-scale density fluctuations using the renormalization group method. Our derivation follows that of *Kichatinov* [1985] (and references therein), who gave a formulation for magnetohydrodynamics and calculated the dissipation coefficients for velocity and magnetic field fluctuations. Our calculation is based on a single-fluid electrostatic model and leads to an expression for the anomalous diffusion of plasma density.

The renormalization group method permits the analysis of nonlinear dynamical systems where perturbation theory is ordinarily inapplicable. Its strategy involves dividing wavenumber space into thin spherical shells, with the presumption that the shells are only occupied out to a finite radius. The dominance of dissipation in the largest of the shells being considered permits the use of perturbation theory there. The method calls for the calculation of the enhanced dissipation due to the ensemble of turbulent wave modes occupying the largest shell. This additional dissipation is seen by the next smaller neighboring shell, which has now been made sufficiently dissipative to permit the use of perturbation theory there. The method is bootstrapping, with the entire spectrum of turbulence ultimately entering into the ensemble averaging. A differential equation can be derived to express the increase of the anomalous diffusivity per increase in unit wavevector space entering into the ensemble. The solution to the differential equation gives the total anomalous dissipation acting upon wave modes lying outside the ensemble.

Using the renormalization group method, we derive below an estimate for the anomalous diffusivity in a two-dimensional ion-neutral collision dominated plasma and apply the result to the case of plasma irregularities in the equatorial F region ionosphere. It will be shown that small- and intermediate-scale plasma waves associated with bottomside spread F scattering layers may inhibit the growth of large-scale plasma waves with wavelengths up to about 100 km. Longer-wavelength waves are already inhibited by finite gradient length scale effects.

Strictly speaking, the renormalization group method applies to turbulent flows. While ionospheric irregularities in the collisional regime cannot be considered turbulent in the Kolmogorov sense, equatorial spread F is a complex statistical flow to which the method is assumed to apply. The small-scale flow will be referred to as turbulent throughout this paper so that the language can be consistent with previous work.

2. Diffusivity Calculation

Here we derive an expression for the anomalous diffusivity in a magnetized ionospheric plasma undergoing a broadband (but band limited) statistical flow starting from the single fluid continuity equation in two dimensions:

$$\frac{\partial n}{\partial t} + \mathbf{v}\cdot\nabla n = D\nabla^2 n \tag{1}$$

$$\mathbf{v} = \hat{b}\times\nabla\phi/B \tag{2}$$

where $\mathbf{v}$ is the guiding center velocity for electrons and ions and $\hat{b}$ is a unit vector in the direction of the geomagnetic field. Let us separate the field quantities into large-scale background components and random small-scale fluctuations

$$n = n_\circ + \delta n, \quad \langle n\rangle = n_\circ \tag{3}$$

$$\mathbf{v} = \mathbf{v}_\circ + \delta\mathbf{v}, \quad \langle \mathbf{v}\rangle = \mathbf{v}_\circ \tag{4}$$

where the angle brackets denote an average over the small spatial scales (with $k \geq k'$, where k' is the smallest wavenumber at which perturbation theory can be directly applied). Averaging the continuity equation over small scales likewise produces a diffusion equation for large scales

$$\frac{\partial n_\circ}{\partial t} + \mathbf{v}_\circ\cdot\nabla n_\circ = D\nabla^2 n_\circ - \langle\delta\mathbf{v}\cdot\nabla\delta n\rangle \tag{5}$$

The goal of this calculation is to assess the impact of the quadratic term on the right side of (5) which represents the effect of the fluctuations on the large-scale flow. We will ultimately show that this term takes the form of an additional diffusion term with a diffusion coefficient that depends on the statistics of the fluctuations.

The smoothed continuity equation above can be subtracted from (1) to yield an equation for the small-scale, fluctuating components of the flow

$$\frac{\partial\delta n}{\partial t} - D\nabla^2\delta n + \delta\mathbf{v}\cdot\nabla\delta n - \langle\delta\mathbf{v}\cdot\nabla\delta n\rangle = -\delta\mathbf{v}\cdot\nabla n_\circ - \mathbf{v}_\circ\cdot\nabla\delta n \tag{6}$$

The left side of (6) has the form of a continuity equation for the small-scale fluctuations. The right side apparently represents perturbations to the fluctuating flow field produced by interactions with the mean (large-scale) flow. This identification motivates the separation of the fluctuation field quantities into two parts: fields associated with the "base turbulence" (described by (6) with zero right side) and perturbations arising from the mean-flow interaction. The base turbulence is decoupled from the large-scale flow. We designate these base turbulence and perturbation components of the fluctuations with the superscripts 0 and 1, respectively.

$$\delta n = n^0 + n^1 \tag{7}$$

$$\delta\mathbf{v} = \mathbf{v}^0 + \mathbf{v}^1 \tag{8}$$

The ordering is such that the perturbation quantities have small amplitudes but are concentrated at the smallest scales being considered.

We neglect the terms in (6) that are quadratic in the fluctuating field quantities or that are quadratic and involve perturbations. We also neglect the partial time derivative by comparison to the Dk^2n^1 term, assuming that the perturbations are dominated by dissipation. This leaves an equation for the perturbed plasma density

$$Dk^2 n^1 = i\int d\mathbf{q}\,\mathbf{v}^0(\mathbf{k}-\mathbf{q})\cdot\mathbf{q}\,n_\circ(\mathbf{q}) + i\int d\mathbf{q}\,\mathbf{v}_\circ(\mathbf{k}-\mathbf{q})\cdot\mathbf{q}\,n^0(\mathbf{q}) \tag{9}$$

in terms of the base turbulence and the large-scale flow. It is written here in the Fourier domain, where products appear as convolution integrals over two-dimensional wavevector space. We implicitly take D here to be a linear operator.

In order to write (9) in closed form, we require a relationship between the plasma density and velocity. This is provided by the condition that the plasma remain charge neutral, or that the current density remain nondivergent. In the collisional regime of the F region ionosphere, the primary current is the ion Pedersen current, given by $\mathbf{J} = ne(\nu_{\text{in}}/\Omega_i B)\mathbf{E}$. (Here ν_{in} and Ω_i are the ion-neutral collision frequency and the ion gyrofrequency, respectively.) Setting the divergence of this current to zero and linearizing about the background plasma density $N_\circ$ and a background zonal electric field $E_\circ\hat{x}$ yields the desired expression for the plasma susceptibility.

$$\phi = -\frac{ik_xE_\circ}{k^2N_\circ}n, \quad \mathbf{v} = \frac{i}{B}\hat{b}\times\mathbf{k}\phi \tag{10}$$

which gives

$$n^1(\mathbf{k}) = \frac{1}{Dk^2}\int d\mathbf{q}M(\mathbf{p},\mathbf{q})\left[n^0(\mathbf{p})n_\circ(\mathbf{q}) + n_\circ(\mathbf{p})n^0(\mathbf{q})\right] \tag{11}$$

$$M(\mathbf{p},\mathbf{q}) = \frac{iE_\circ}{2BN_\circ}\left(\frac{p_x}{p^2}-\frac{q_x}{q^2}\right)(\mathbf{p}\times\mathbf{q})\cdot\hat{b} \quad (12)$$

in terms of the auxiliary wave vector $\mathbf{p}=\mathbf{k}-\mathbf{q}$ and the three-wave coupling coefficient M [*Kintner and Seyler*, 1985].

What remains is to substitute (11) into (5) to assess the impact of the small-scale fluctuations on the large-scale flow. We are only interested in terms which involve perturbations, since the base turbulence is decoupled from the large-scale flow. However, we will neglect terms that are quadratic in the perturbations. Therefore

$$\langle\delta\mathbf{v}\cdot\nabla\delta n\rangle = \langle\mathbf{v}^1\cdot\nabla n^0\rangle+\langle\mathbf{v}^0\cdot\nabla n^1\rangle \quad (13)$$

The two terms on the right side of (13) can be shown to make identical contributions, owing to the symmetric construction of the M operator and the relationship between velocity and density perturbations. Using the same coupling coefficient shorthand that led to the derivation of (11), it can further be shown that

$$\langle\mathbf{v}^0\cdot\nabla n^1\rangle = -\int d\mathbf{k}d\mathbf{p}\, M(\mathbf{p},\mathbf{q})M(-\mathbf{p},\mathbf{k}) \cdot\frac{2}{D(q)q^2}\langle|n^0(\mathbf{p})|^2\rangle n_\circ(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}$$

in which $\mathbf{p}+\mathbf{q}=\mathbf{k}$ form another wave triplet and where we have assumed that the various wave modes making up the base turbulence are statistically uncorrelated, so that $\langle n^0(\mathbf{p})n^0(\mathbf{q})\rangle = \langle|n(\mathbf{p})|^2\rangle\delta(\mathbf{p}+\mathbf{q})$.

At this point, the integral in (14) can be simplified with an assumption regarding the form of $n_\circ$, the large-scale density distribution. Since we are ultimately concerned with calculating the anomalous dissipation seen by horizontally propagating large-scale ionospheric plasma waves, $n_\circ$ will make the greatest contribution to the integral when $\mathbf{k}=k\hat{x}$. Furthermore, since $\mathbf{k}$ is a wave vector of the large-scale flow and $\mathbf{p}$ and $\mathbf{q}$ are wavevectors of the small-scale fluctuations, we expect the most significant contributions for $p\sim q\gg k$. Under these circumstances, the product of the coupling coefficients in (14) can be approximated by

$$M(\mathbf{p},\mathbf{q})M(-\mathbf{p},\mathbf{k}) \approx -\left(\frac{E_\circ}{2BN_\circ}\right)^2\frac{p_z^2}{p^2}k^2 \quad (14)$$

where additional terms that vanish under the $\mathbf{p}$ integration have been neglected. The inherent anisotropy in the problem can be understood by considering that the plasma waves in question are transverse, so that vertically propagating fluctuations will tend to cause the greatest dissipation of the horizontal density gradients set up by horizontally propagating large-scale waves. (The fact that waves with wavevectors set at right angles couple the most strongly is a consequence of the form of the mode coupling coefficient M and has been discussed by *Zargham and Seyler* [1987].) With the k dependence of the coupling coefficients explicitly extracted, the inverse Fourier transform in (14) may new be performed, resulting in a term that is the Laplacian of the large-scale density field.

With a change of variables from p to k, equation (13) can now be written in the form of a diffusion operator on the large-scale density field $n_\circ$. The diffusion coefficient is expressed as an integral over wavenumbers with $k\geq k'$. It will be insightful to break down the integration space into a series of wavenumber shells, denoted below by Δk.

$$\langle\delta\mathbf{v}\cdot\nabla\delta n\rangle = -\int_{\Delta k} d\mathbf{k}\left(\frac{E_\circ}{BN_\circ}\right)^2\frac{k_z^2}{k^2}\frac{|n(\mathbf{k})|^2}{Dk^2}\nabla^2 n_\circ (15)$$

so that

$$\partial D/\partial\Delta k = \left(\frac{E_\circ}{BN_\circ}\right)^2\frac{k_z^2}{k^2}\frac{|n(\mathbf{k})|^2}{Dk^2}$$

Since the fluctuations are assumed to be band limited, there is some largest wavenumber shell Δk making a contribution to the integral above. The differential increase to the integral made by that shell per unit area of wave vector space is denoted $\partial D/\partial\Delta k$. D in the integrand in this case would just be the ambipolar diffusion coefficient. However, the fluctuations in the wavenumber shell immediately inside the first would be affected by the increased diffusion coefficient just inferred, and so on down to the smallest wavenumber shell associated with the fluctuations. Equation (15) therefore implies a differential equation for $D(\Delta k)$, which can be readily solved. The total anomalous diffusion coefficient arising from fluctuations with wavenumbers greater than k may then be written as

$$D^2(\tilde{k}) = 2\left(\frac{E_\circ}{BN_\circ}\right)^2\int_k^\infty d\mathbf{k}\frac{k_z^2}{k^2}\frac{|n(\mathbf{k})|^2}{k^2}+D^2(\infty)(16)$$

This appears as the diffusion coefficient in the original continuity equation for the large-scale flow (5), assumed to be contained entirely in the sphere of wavenumbers with $\tilde{k}<k$. $D(\infty)$ above is a boundary condition equal to the ambipolar diffusion coefficient.

3. Application: Equatorial Spread F

Let us apply this result to the problem of plasma irregularities in the equatorial F region ionosphere associated with equatorial spread F. Such irregularities are known to take on scale sizes between centimeters and hundreds of kilometers [*Kelley*, 1989]. Much of our knowledge about these irregularities has come from coherent scatter radar observations made at the magnetic equator, notably at the Jicamarca Radio Observatory near Lima, Perú. *Woodman and La Hoz* [1976] identified four types of radar backscatter morphologies that occur during spread F conditions, two of which being bottomside spread F and topside radar plumes. The former is a reference to layers of intermediate-scale plasma structuring observed mainly on the bottomside of the

F layer, and the latter to large-scale plasma depletions that penetrate well into the topside ionosphere. Both of these phenomena are associated with the ionospheric interchange instability, which has been investigated in numerous computational studies [*Zalesak and Ossakow*, 1980; *Keskinen et al.*, 1980; *Zargham and Seyler*, 1989]. Both are observed by VHF and UHF coherent scatter radar, indicating that the underlying plasma instabilities are intense enough to drive small-scale, linearly stable plasma waves that give rise to coherent radar scatter.

Bottomside spread F produces a spectrum of plasma waves with horizontal scale sizes seldom much larger than the horizontal dimension of the scattering volume of the Jicamarca radar, which is effectively 10-20 km at F region heights. (The phase fronts of plasma waves in bottomside layers can sometimes be resolved distinctly in radar scattered power time histories, although usually they cannot.) Bottomside layers seem to be a manifestation of the collisional interchange instability operating at intermediate scale sizes, where the linear growth rate of the instability is greatest [*Zargham and Seyler*, 1987]. In contrast, topside radar plumes are very large-scale phenomena, with horizontal scale sizes of several hundred kilometers.

We wish to estimate the effect of intermediate- and small-scale (10 km $> \lambda >$ 100 m and $\lambda <$ 100 m) waves in bottomside spread F upon large-scale ($\lambda >$ 10 km) plasma instabilities. Computer fluid simulations of the ionospheric interchange instability suggest that the growth of large-scale waves is inhibited to some degree by the presence of preexisting intermediate-scale bottomside waves. When the intermediate-scale waves are allowed to grow from initial broadband seed noise, large-scale waves that would otherwise tend to grow do not. (S. Zargham, personal communication, 1990.) Recent experimental observations made at Jicamarca confirm that large-scale radar plumes seldom occur once bottomside spread F has developed [*Hysell and Burcham*, 1998]. These findings can, in part, be explained by the fact that the intermediate-scale waves efficiently dissipate the background vertical plasma density gradient necessary to drive the interchange instability. However, anomalous diffusivity, generated by the small- and intermediate-scale waves and seen by the large-scale waves, may also play a role.

Figure 1 shows plasma density data taken by the retarding potential analyzer aboard the AE-E satellite during a pass through bottomside spread F plasma irregularities. (W. B. Hanson, personal communication, 1993.) They show 3-s time intervals of density measurements. The power spectra of the plasma density structures are characteristically flat at long wavelengths and adopt power law scaling with a -2 spectral index (approximately) at short wavelengths. The spectral break occurs at a wavelength ($2\pi/k_1$) anywhere between 500 m and 5 km but usually between 1 and 2 km. (We assume here that the flow is frozen in, so that the time axis of the density data can be converted to a spatial scale.)

The spectrum of bottomside spread F may therefore, on the basis of the in situ data, be modeled approximately as

$$s_1(k) = \frac{2\langle \Delta N^2 \rangle / k_1}{1 + (k/k_1)^2} \tag{17}$$

$$\rho_1(x) = \langle \Delta N^2 \rangle e^{-k_1|x|} \tag{18}$$

where $s_1(k)$ is the one-dimensional spectral density of the irregularities with mean-squared amplitude $\langle \Delta N^2 \rangle$. We are referring to the spectrum one would calculate from the time series data obtained from a satellite pass through the irregularities, for instance. Here $\rho(x)$ is the corresponding one-dimensional autocorrelation function of the density irregularities. Our preceding calculation requires us to specify the complete two-dimensional spectrum of the bottomside spread F irregularities. For the sake of expediency, we merely assume here that the irregularities are statistically isotropic. We then compute the corresponding two-dimensional irregularity spectrum by writing the axisymmetric form of the two-dimensional autocorrelation function and Fourier transforming (see *Fredricks and Coroniti* [1976] or *Woodman and Basu* [1978]):

$$\rho_2(r) = \langle \Delta N^2 \rangle e^{-k_1 r} \tag{19}$$

$$s_2(k) = \frac{2\pi \langle \Delta N^2 \rangle k_1}{(k_1^2 + k^2)^{3/2}} \tag{20}$$

Substituting this spectral form into (16) leads to the following anomalous diffusion coefficient estimate.

$$D(\tilde{k}) \approx \left\langle |\Delta N/N_\circ|^2 \right\rangle^{1/2} \left(\frac{E}{B} + \frac{g}{\nu_{\text{in}}} \right) \cdot \frac{2\pi}{k_1} \left(\ln(k_1/k_\circ) + \ln 2 - 1 \right)^{1/2} \tag{21}$$

where we have made use of the fact that $k_1 \gg k_\circ$, $k_\circ$ being the lower cutoff of the irregularity spectrum, and have generalized the earlier result by including currents driven by gravity alongside the background ionospheric Pedersen current driven by $E_\circ$. The wavenumber $\tilde{k}$ is the wavenumber of the large-scale waves, associated in this case with radar plumes, where it has been assumed that $k_\circ \gg \tilde{k}$. The strong dependence of our estimate on the wavenumber of the spectral break in our model is a consequence of the k^{-2} factor in (16), larger-scale fluctuations being more efficient dissipators of plasma structure. We note that, while our model suffers from some uncertainty regarding the exact value of the outer scale of bottomside spread F, this parameter only enters the final estimate logarithmically.

For example, assuming $k_\circ = 2\pi/20$ km, $k_1 = 2\pi/2$ km, $(E/B + g/\nu_{\text{in}}) = 40$ m/s, and and RMS density fluctuation level of 0.5, we can estimate an anomalous diffusion coefficient $D \approx 5.7 \times 10^4$ m^2/s, which is about 5 orders of magnitude greater than the expected am-

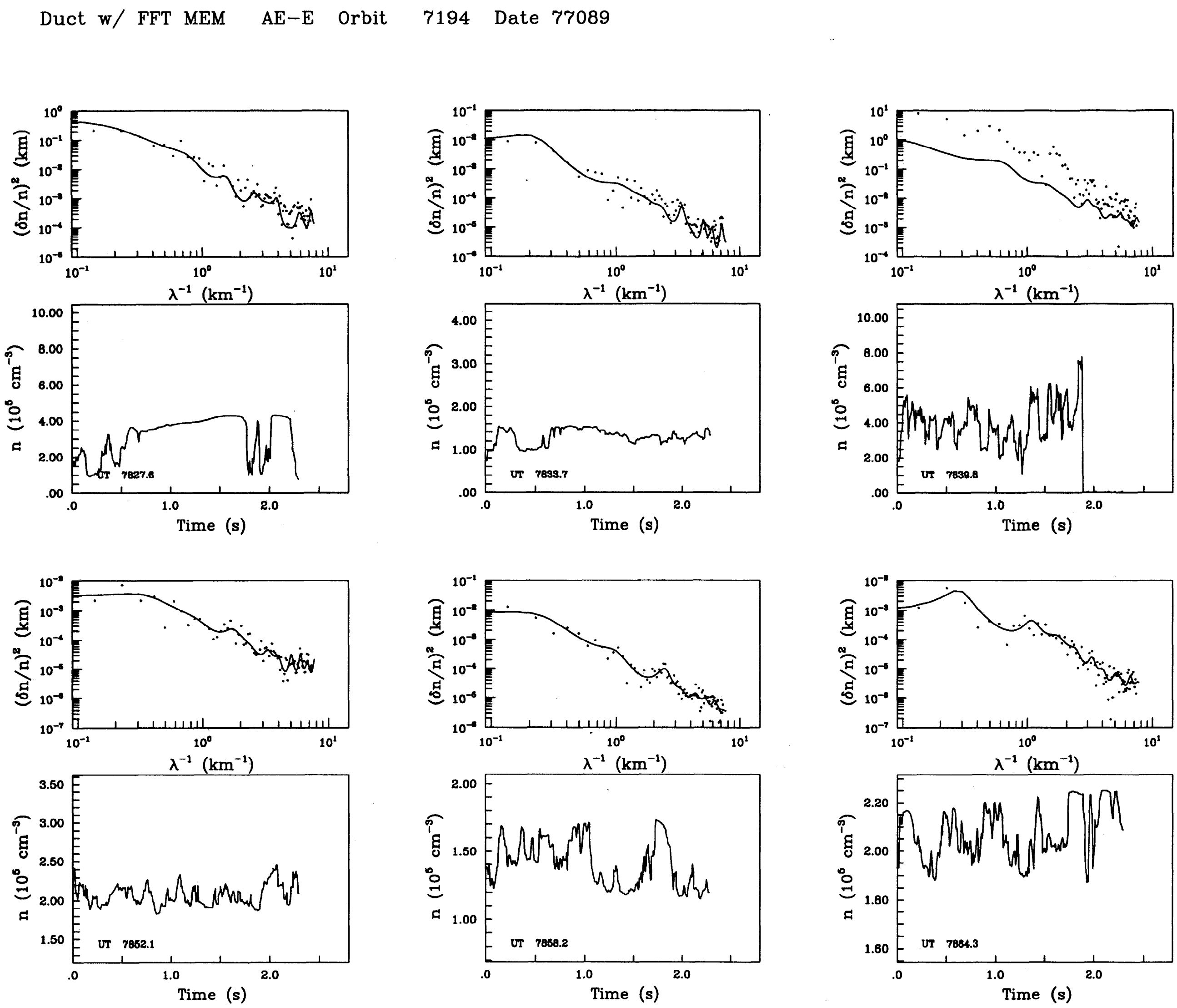

Figure 1. Observations of plasma density irregularities in bottomside spread F made by the AE-E spacecraft. The plots with linear scales show three seconds of time series data. The plots directly above each of those with log scales represent the associated spectral density. Both periodograms (points) and MEM spectra (solid lines) are shown. The horizontal axes on the spectral plots represent inverse wavelength.

bipolar diffusion coefficient. In comparison, the linear, nonlocal growth rate of the large-scale collisional interchange instability can be expressed as

$$\gamma(k) \approx \frac{1}{L}\left(\frac{E}{B}+\frac{g}{\nu_{\text{in}}}\right)\left(1-\frac{1}{kd}\right)-Dk^2 \quad (22)$$

where L is the background density gradient scale length and d is a parameter which specifies the vertical extent of the positive vertical density gradient [*Zargham and Seyler*, 1989]. (We have taken the limit $kd \gg 1$ here for simplicity.) Long-wavelength waves with $\lambda/d \gtrsim 1$ are clearly stabilized by nonlocal effects. Meanwhile, dissipative damping due to the wave-driven diffusion found above is significant when $\lambda \lesssim 100$ km. Given $L = 25$ km and $d = 50$ km, the new dissipation term in fact introduces a peak in the linear growth rate at a wavelength of very nearly 100 km. The growth rate of this fastest growing large-scale wave is approximately half the linear growth rate in the local, dissipation-free limit.

The finding of an induced long-wavelength peak in the linear growth rate may explain why it is that bottomside layers very often launch small plasma depletions into the topside ionosphere at regular spatial intervals of approximately 100 km. (These depletions look like mini radar plumes in the radar data, with radar echoes that last only as long as would take a small scattering target to drift through the radar beam.) If the height of the bottomside layer were to be modulated by a large-scale plasma wave, the resulting crests would be the most likely sites for mini plumes to form. Meanwhile, the preceding calculation does not support the notion than bottomside spread F suppresses the occurrence of large-scale ($\lambda \gtrsim 100$ km) topside waves and radar plumes in nature. However, intermediate-scale bottomside waves may nonetheless inhibit the growth of large-scale waves by rapidly dissipating the steep bottomside F region plasma density gradient necessary for instability.

4. Summary and Analysis

We have derived an expression for anomalous diffusivity in a magnetized plasma driven by small-scale turbulence. The derivation assumes that Pedersen currents are the dominant currents, or that $\Omega_e, \Omega_i \gg \nu_{\text{in}} \gg \omega$, assumes a zonal background electric field (or vertical gravity), and also assumes that the large-scale waves of interest propagate preferentially in one direction (horizontally).

It is informative to compare (16) to the expression obtained by *Montgomery* [1972], who calculated the anomalous diffusivity in a bounded, strongly magnetized, two-dimensional guiding center plasma using a test particle approach. His calculation assumed like ours that the amplitudes of the wave normal modes were statistically uncorrelated and further assumed that the test particle positions were uncorrelated with those of the background particles. The diffusivity was expressed in terms of a discrete sum over the modal electric field amplitudes. Converting those electric field amplitudes to densities with the help of (10) and taking the continuous limit reproduces our expression (16), except with the k_z^2/k^2 factor replaced by k_x^2/k^2. The discrepancy is due to the fact that the two derivations answer somewhat different questions. The *Montgomery* [1972] derivation predicts the total diffusivity in a plasma driven by all the wave modes present. The k_x^2/k^2 factor expresses the fact that horizontally propagating waves generate the largest electric fields and the greatest plasma advection and transport by the requirements of quasi-neutrality. Meanwhile, our calculation involves the wave-wave coupling between small-scale fluctuations and a large-scale, horizontally propagating wave. This coupling is most efficient if the fluctuations propagate vertically, hence the k_z^2/k^2 factor.

We have shown that fully developed bottomside spread F waves may inhibit the large-scale collisional interchange instability via anomalous dissipation and suppress waves with wavelengths less than about 100 km. Combining this effect with finite gradient length scale stabilization produces a peak in the linear growth rate of the collisional interchange instability that may explain the 25-100 km spatial periodicity of high-altitude depletions that emerge from bottomside spread F layers as observed by ground-based radar [*Hysell and Burcham*, 1998]. Anomalous dissipation does not seem to be strong enough to suppress the very large-scale depletions underlying the largest radar plumes observed with ground-based radar. The fact remains that such radar plumes seldom occur once fully developed bottomside spread F has emerged on a given evening.

The present analysis has taken for granted the form of the spectrum of the ionospheric irregularities without considering the processes responsible for generating them. Recently, J. P. Flaherty et al. (Large-amplitude transient growth in the linear evolution of equatorial spread F with a sheared zonal flow, submitted to *Journal of Geophysical Research*, 1998) analyzed the linear collisional interchange instability, taking into account the effects of shear, a seemingly ubiquitous property of bottomside spread F layers. They found that shear causes the system to become highly nonnormal, permitting a strong transient response not predicted by a conventional eigenmode analysis. The transient response is seen principally at wavelengths near the marginal wavelengths of the unsheared system, which are stable in the sheared system. As time progresses, the response occurs at increasingly long wavelengths, including wavelengths longer than the gradient scale length. The transient response eventually decays, and the system becomes dominated by the long wavelengths predicted by eigenmode analysis. It is at this time that topside depletions are thought to be produced by the nonlinear instability and also that the present analysis begins to have a clear bearing on the wavelengths of the irregularities that result. The anomalous diffusivity we have found would increase the wavelengths of the fastest growing eigenmodes, explaining the large-scale periodicity of topside depletions rising out of bottomside layers. It is not so clear at this point what effect the anomalous diffusivity would have on the transient response itself, which may persist for some time in the early evening.

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