Performance of the S-band stratospheric decoding system at the Arecibo Observatory

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I . Abstract

The objective of our work is to propose and analyze a decoding procedure useful to achieve unprecedented spatial and temporal resolution in CW S-band radar measurements at the AO. We find that using hard limiting transformations of the detected echoes (to achieve computational speed) does not distort significantly the process of spectral estimation. Moreover our procedure is efficient in the sense that statistical accuracy of individual estimates is kept with small increase in (coherent) integration time specially if the time series is oversampled. These characteristics make our estimation method very convenient.

II . Introduction

There is a real need for a detailed spatial and temporal description of the atmosphere. This need has been defended by several workers of the different regions of the atmosphere but in particular, for the stratosphere, by Woodman R.F. (1980). A problem that we are interested is the description of the morphology of turbulence in the lower stratosphere and its contribution to the diffusion characteristics in the region. To accomplish our goals we require continuous observations of the medium using tools that are capable of achieving spatial and temporal resolutions of,say 15 meters and 10 seconds respectively. At the Arecibo observatory the S-band radar system (2380 Mhz), used in a bistatic fashion (because it lacks a T/R switch), has the potential to be used for our scientific goals. In this paper we propose and evaluate a procedure that is being successfully used in the lower atmosphere . Some preliminary results have already been presented by Ierkic H.M. (1987),and more will be shown, in another work, at this workshop. The next section is dedicated to the presentation of the theory and later we close pointing the important results of our study as well as the orientation that our effort is taking. We want to state here that the linear characteristic of the calculated transfer function has been verified under laboratory conditions.

III. Theory and discussion

Consider, for the purposes of this work that a CW coded signal is being scattered in the stratosphere and that it is being received and demodulated to base band. Furthermore, assume that the specific code used is a pseudo-noise one (MacWilliams and Sloane, 1976) and that the scattering medium has large correlation times relative to the duration of the code. The detected signal contaminated by diverse sources of noise is first digitized and then decoded using a correlator. We will present results for the cases when the echoes are digitized using 1 bit, 1.6 bits (3 levels) and many bits; the discussion will concentrate in the 3 levels case for the algebraic development that follows.

Assuming that there is only a single scattering target, the action of the decoder is described by the following formula,

$$e(k)=SUM\{g(Vs*c(i)+Vn(i))*c(i+k-1)\}, \quad i=1,2,...,l*N. \qquad (1)$$

Here, N is the number of bauds of the code, l the number of times the code is recycled before the coherent integration time is reached, Vs represents the intensity of the echo, $Vn(i)$ is the i-th noise sample, $c(i)$ is the i-th value of the code (either -1 or $+1$), g describes the transfer function characteristic of the digitizer and $e(k)$ is the decoded signal corresponding to lag k with the idealized scatterer located at $k=1$. The noise samples have an underlying Gaussian distribution with probability density function f and distribution function P. The expected value of the output given by the correlator is,

$$Mu(k)=E[e(k)] \qquad (2)$$

Equation (2) can be readily evaluated to give,

$$Mu(1)=p0+2*p1-1. \qquad (3a)$$

$$Mu(k)=-Mu(1)/N \qquad (3b)$$

where,

$$p0=P(Vc-Vs/Vn)+P(Vc+Vs/Vn)-1. \qquad (4a)$$

$$p1=1.-P(Vc-Vs/Vn) \qquad (4b)$$

with Vn denoting the standard deviation of the noise and Vc the threshold level of the digitizer. Equation (3) shows that in order to keep range contamination low the length of the pseudo noise (PN) sequence should be as long as possible.

Figure 1 illustrates the behavior of $Mu(1)$ versus Vs keeping Vc as a parameter and the several curves can be understood intuitively.

In order to evaluate the performance of the proposed decoding procedure we find convenient to calculate the variance of the statistical estimator $e(k)$ in the usual way,

$$Var(k)=E[e(k)**2]-Mu(k)**2 \qquad (5)$$

To actually compute the variance we assume that the noise samples are uncorrelated and use the property of the PN sequences that out of the N bauds of the code, $(N+1)/2$ are minus one and the rest plus one. Explicit computation reduces (5) to

$$Var(k)=(1.-p0-(1.-p0-2.*p1)**2)/(l*N) \qquad (6)$$

The measure of performance that we use can now be formulated by requiring to find for each Vs/Vn the value of Vc/Vn that minimizes the modified variance defined as $MVar(k)$ below, MVar(k)=(Var(k)/Mu(k)**2)*(Vs/Vn)**2 (7)

the result of the optimization procedure is presented in figure 2 and can be summarized by saying that for small values of Vs/Vn (the actual situation in our experiments) the optimum value of Vc/Vn is about 0.60. Moreover by comparing the performance of the three level scheme with the multibit case (when, Mu(1)=Vs, Mu(k)=-Mu(1)/N and Var(k)=Mu(k)**2/1N) we can see from figure 2 that the relative efficiency turns out to be about 0.81. This last result implies that with about 25% longer coherent integration time we achieve the same accuracy as in the many bit case.

Another very encouraging property of the scheme we are considering is that the decoded signal does not show appreciable distortion over a wide range of values of Vs/Vn, including those found under practical circumstances. To see this linearity property we use a Taylor expansion of Mu(1) to get,

Mu(1)=
2.*f(Vc/Vn)*(Vs/Vn)*{1.+0.167*((Vc*Vs)/(Vn**2))**2} (8)

with Vs/Vn small and the quadratic term contribution minor. The similar expansion for the variance is,

Var(k)=
{2.*(1.-P(Vc/Vn))+f(Vc/Vn)*(Vs/Vn)**2*(Vc/Vn-4.*f(Vc/Vn))}/1*N (9)

The description of the 1 bit case (2 level digitizer) can be obtained by simply setting Vc=0 in the equations above, so that in particular we are ready to get the result that this decoding scheme has a performance such that it requires about 1.6 times longer to get the same accuracy as in the multibit case.

Another interesting property to be evaluated is the improvement that oversampling (i.e. the noise samples become correlated) will bring about. The analysis now is again simple but cumbersome and we will just state some results; before, we assume that the noise has flat spectral characteristics a fact that can be altered with no major consequences. It can the be shown that for the multibit case the variance is given by

Var(k)=
2.*Vn**2/(M**2*1*N)*{M/2.*r(0)+(M-1)*r(1)+ ... +r(M-1)} (10)

with M representing the degree of oversampling (e.g. M=2 is twice the Nyquist rate) and r(k)=E[Vn(i)*Vn(i+k)]. Evaluating (10) for M=2 and M=1 we find that there is an improvement in the variance of about 0.82. For M large this improvement asimptotically reaches a value of about 0.77 which normally does not justify going beyond M=2. The variance for the 1.6 bit case with oversampling can be shown to be,

Var(1)=
1/(M*1*N)**2*[2*1*N*{M/2*Re(0)+(M-1)*Re(1)+ ... +Re(M-1)}+
{1*(N-1)/2}*{(M-1)*Re(M-1)+ ... +Re(1)}+{1*(N+1)/2-1}*{(M-1)*
Ru(M-1)+ ... +Ru(1)}-{1*N*M**2+(1N-1)*M*(M-1)}*Mu(1)**2] (11)

where,

Re(k)=E[g(Vs+Vn(i))*g(Vs+Vn(i+k))] (12a)
Ru(k)=E[g(Vs+vn(i))*g(Vs-Vn(i+k))] (12b)

and specific properties of the PN codes were used to get (11). Equation (11) is cumbersome to work with analitically, however if we consider $Vs=0$, then we can use the results obtained by Hagen and Farley (1973). In particular, using their figure 7 (curve 11) to find $Re(k)$ (note $Ru(k)=-Re(k)$ when $Vs=0$), we can readily conclude that with oversampling the performance of the three level scheme is at least as good as the many bit case with no oversampling.
A final point is that the correlator at the AO is peculiar in that, first it does not reset the 6 least significant bits of its accumulators and second it drops those bits at the end of the operation. We have verified that this fact will cause an error of plus or minus one in the final count and turns out to be unimportant within the range of values used in the experiments.

IV . Conclusions

We have shown that within a wide margin of values of Vs the estimator (1) is linear, consequently we can say that our statistical procedure introduces no distortion in the measurements. We have also shown that the method in consideration (1 bit, 1.6 bits) is fairly efficient when compared with the multibit case specially if the random series is oversampled (in the sense that the points are correlated). It is also worth mentioning that crude sampling makes it possible to achieve fast computational speeds which in turn imply that we can do our atmospheric studies with unprecedented height resolution. A simple extension of the theory presented shows that in the presence of a continuum of scattering (a scenario closer to the actual physical situation) the measured quantity will be,

$$Mu(k)= 2.*f(Vc/Vn)*(Vs(k)/Vn)-2.*f(Vc/Vn)*\{SUM(Vs(j)/Vn)\}/N \qquad (13)$$

for the three level case, with the index j s.t. $1=< j =< N$ and j different than k. In the multibit case the expression reads,

$$Mu(k)=Vs(k)-SUM(Vs(j)/Vn)/N \qquad (14a)$$

$$Var(k)=Vc**2/1*N \qquad (14b)$$

We are proceeding forward with the analysis to consider more realistic conditions (instrumental and scattering models). We also want to carry on, the analysis considering the codes as random variables (here we regarded them in a strictly deterministic way) and would like to find further common grounds with the work of Hagen J.B. and D.T. Farley (1973). Finally, we are exploring the application of the technique to measurements in the presence of strong reflections or clutter.

V . Acknowledgements

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VI . References