An Analysis Technique for Deriving Vector Winds and In-Beam Incidence Angles from Radar Interferometer Measurements

M. F. LARSEN,* R. D. PALMER, S. FUKAO, R. F. WOODMAN,** M. YAMAMOTO, T. TSUDA, AND S. KATO

Radio Atmospheric Science Center, Kyoto University, Uji, Japan

(Manuscript received 29 October 1990, in final form 8 April 1991)

ABSTRACT

We present a method for deriving horizontal velocities, vertical velocities, and in-beam incidence angles from radar interferometer data. All parameters are calculated from the slope and intercept of straight lines fitted in a least-squares sense to the variation of the signal phase as a function of radial velocity for each pair of receiving antennas. Advantages of the method are that the calculations are computationally fast and simple, and the analysis leads to relatively simple expressions for the uncertainty in the velocity measurements.

1. Introduction

The radar interferometer technique has not seen widespread use in the field of radar meteorology, but the results of a number of studies have been described in the literature (e.g., Farley et al. 1981; Röttger and Ierkic 1985; Adams et al. 1986; Kudeki 1988; Kudeki et al. 1989; Röttger et al. 1990; Palmer et al. 1990). Most applications of the method have focused on locating localized scatterers within a larger volume sampled by the radar. A by-product of the analysis is usually the velocity of either a single scatterer or a number of scatterers. A minimum requirement for interferometry is that there must be at least two spatially separated receiving antennas, although three antennas are preferable so that two nonparallel baselines can be formed. The imaging Doppler interferometer technique developed by Adams et al. (1986) uses many receiving antennas along each of two baseline directions. The phase progression in the signals across the array is used to create a map of the location of the scatterers in each range resolution cell. The radial velocities of at least three scatterers are then combined to give the three-dimensional velocity for a given height. Such a system may have potential as an operational wind-profiling system, but the complexity of the hardware, software, and the interpretation of the results are hurdles.

The spaced antenna method for deriving winds has been applied to measurements made with a number

of different research radars (e.g., see Larsen and Röttger 1989; and the references therein). The setup used in the spaced antenna method is for all practical purposes identical to the interferometer setup; that is, one transmitting array and a minimum of three receiving antennas deployed along two nonparallel baselines. As a result, there has been considerable confusion about the differences between the two methods. Early spaced antenna measurements utilized only the correlation between the received power, but later systems use the correlations between the complex time series so that the use of the signal phase cannot be applied as a clear discriminator between the two techniques. A possible distinction between interferometry and spaced antenna analysis may be that the latter deals with the macroscopic velocity appropriate to the sampling volume as a whole, whereas the former deals with the microscopic velocities associated with structure within the sampling volume. Nonetheless, confusion is warranted.

There has been ongoing discussion in the literature (e.g., see Larsen and Röttger 1989) about the relationships between the so-called Doppler wind measurement technique, in which the radial velocities from three or more beam directions are combined to yield the vector velocity, and the spaced antenna method, in which the winds are determined from measurements of the velocity with which the turbulent structure drifts across the array of receiving antennas, as indicated by the times when the peaks in the cross correlation of the received signals occur. Briggs (1980) first showed that there is a certain equivalence between the Doppler and spaced antenna techniques. More recently, Liu et al. (1990) and Van Baelen (1990) have shown that there is an exact equivalence between the spaced antenna and interferometer techniques, although the former is carried out in the time domain while the latter deals with Doppler velocities and is carried out in the fre-

* On leave from the Department of Physics and Astronomy, Clemson University.

** On leave from Instituto Geofísico del Perú, Lima, Perú.

Corresponding author address: Dr. Miguel F. Larsen, Dept. of Physics and Astronomy, Clemson University, Clemson, SC 29631.

quency domain. The result is perhaps not surprising when the similarity in the instrumental setup for the two methods is considered.

One result of the analyses by Liu et al. (1990) and Van Baelen (1990) was that they found the horizontal velocity is related to the slope of the linear variation of the difference in phase as a function of the radial Doppler velocity of the signals in two adjacent receiving antennas. The results suggested that the slope of a line fitted to the measured phase would give an alternative measure of the velocity, as opposed to determining maxima in the cross-correlation functions. The original idea, however, is not entirely attributable to either Liu et al. (1990) or Van Baelen (1990). Similar or related methods for determining the velocities have been discussed by other authors in connection with interferometry studies (Farley et al. 1981; Röttger and Ierkic 1985; Adams et al. 1986; Kudeki 1988; Kudeki et al. 1989; Röttger et al. 1990), although usually in the context of single scatterers. A further limitation of the equations presented by Liu et al. (1990) was that they applied only to a two-dimensional case, and some other effects related to aspect sensitivity were omitted from the analysis. Our purpose is to present a practical method for carrying out the interferometer analysis that leads to the vector horizontal winds, true vertical velocities, in-beam incidence angles, and error bar estimates. Off-vertical in-beam incidence angles can be introduced in the measurements when aspect sensitivity is important and, in particular, when the aspect sensitive layers are inclined with respect to the horizontal. Larsen and Röttger (1991) have dealt extensively with the latter effect. The essence of the method is to determine the arrival angle of signals within the beam from the phase difference between signals received in spatially separated receiving antennas. The change in the phase difference as a function of the radial Doppler velocity can be used to determine the horizontal winds. The value of the phase difference when the radial Doppler velocity is zero will depend on both the off-vertical incidence angle and the true vertical velocity. All the parameters can be derived from the phase of the coherence function as we will show. Our assumptions and approximations will be suitable for conditions characteristic of tropospheric and to some extent lower stratospheric flows.

Van Baelen (1990) and Van Baelen and Richmond (1990) have addressed some of the same questions that we address here. Our work has progressed in parallel with theirs and, although there is general agreement between the two sets of results, the approaches are somewhat different.

In the next section, the basic equations are derived. Section 3 uses sample data obtained with the MU radar located in Japan to illustrate the application of the method. Section 4 discusses the practical aspects of applying the method. Section 5 contains the conclusion.

2. Derivation of the equations

a. The case without aspect sensitivity

Given two spatially separated receiving antennas, the angle of arrival of the received signals in the plane defined by the antenna baseline and the vertical direction can be determined from the phase difference between the signals in the two antennas. Specifically, the phase difference ϕ_{ij} between the signals in the i th and j th receiving antennas will be given by

$$\phi_{ij} = kd_{ij} \sin \delta', \quad (1)$$

where d_{ij} is the distance separating the antennas, and $k = 2\pi/\lambda$ is the wavenumber, with λ being the radar wavelength. The angle δ' is measured with respect to zenith and defines the incidence angle in the vertical plane along the antenna baseline. Figure 1, taken from Larsen and Röttger (1991), illustrates the geometry. The relationship given in Eq. (1) is used in all interferometer analyses and has been presented or derived in a number of articles (e.g., Adams et al. 1986).

The horizontal area illuminated by the transmitted beam can be represented as in the schematic shown in Fig. 2. The x' axis is parallel to the receiving antenna baseline with positive x' along the azimuth α . The y' axis is perpendicular to the antenna baseline. The horizontal wind is in the direction defined by the azimuth angle θ shown by the heavy arrow. We define a rotated coordinate system such that x is parallel to the wind direction and y is perpendicular to the wind direction. All azimuths are measured with respect to north, although the northward direction is not shown in the diagram in Fig. 2. The radial velocity V_r measured at any point within the beam will be given by

FIG. 1. Diagram showing how a phase difference is produced in two adjacent receiving antennas by asymmetric scattering within the beam.

FIG. 2. Diagram showing contours of constant radial velocity (light solid lines) within the horizontal plane illuminated by the transmitted beam. The receiving antennas are located along the x axis, which corresponds to an azimuth measured from north of α . The wind direction is indicated by the heavy arrow that points in the azimuth direction θ .

$$V_r = V_h \cos \mu \sin \zeta + w \cos \zeta, \quad (2)$$

where μ is the angle between the x axis and the position vector of the point in the horizontal plane. Here V_h is the magnitude of the horizontal wind, ζ the zenith angle, and w the vertical velocity component. Equation (2) is valid for any point within the illuminated volume; that is, any point within the outer circle in Fig. 2, so that μ will vary from 0° to 180° , and ζ will vary from 0° to the beamwidth value for the range of points within the beam. We will assume that the horizontal and vertical velocities are uniform throughout the illuminated volume. The zenith angles considered throughout the paper are small since the measurements are all within the width of the vertically pointing beam. Therefore,

$$\cos \zeta \sim 1. \quad (3)$$

We can replace $\cos \mu$ and $\sin \zeta$ with functions of x and y to get

$$\begin{aligned} V_r &= V_h \frac{x}{(x^2 + y^2)^{1/2}} \frac{(x^2 + y^2)^{1/2}}{z} + w \\ &= V_h \left(\frac{x}{z} \right) + w \end{aligned} \quad (4)$$

where z is the altitude of the scattering volume. Since z , V_h , and w are constant for a given range resolution cell, lines of constant x are also lines of constant radial velocity. The latter are indicated in Fig. 2 by the light solid lines perpendicular to the heavy arrow representing the wind direction.

The Fourier transform of the time series of voltages measured with each of the receiving antennas leads to a representation of the signals as an amplitude and phase for each Doppler frequency ω . Details can be found in any number of basic texts, including Doviak and Zrnić (1984). We can represent the result of the Fourier transformation as

$$F(\omega) = A(\omega) \exp[i\phi(\omega)] \quad (5)$$

where $A(\omega)$ is the amplitude and $\phi(\omega)$ is the phase. The squared magnitude of $F(\omega)$ is the standard Doppler spectrum. With two receiving antennas, we have

$$F_1(\omega) = A_1(\omega) \exp[i\phi_1(\omega)] \quad (6a)$$

$$F_2(\omega) = A_2(\omega) \exp[i\phi_2(\omega)]. \quad (6b)$$

The cross spectrum is then defined as

$$\begin{aligned} C_{12} &= F_1(\omega) F_2^*(\omega) \\ &= A_1(\omega) A_2^*(\omega) \exp\{i[\phi_1(\omega) - \phi_2(\omega)]\} \end{aligned} \quad (7)$$

where $*$ represents the complex conjugate. The amplitude of C_{12} as a function of frequency is effectively the same as the amplitude of the Doppler spectrum, since F_1 and F_2 both represent the Fourier transforms of the signals received with vertically pointing transmitting and receiving beams. The phase of the cross spectrum represents the phase difference between the signals received in adjacent receiving antennas as a function of Doppler shift or radial velocity.

The cross spectrum produces a Doppler sorting of the contributions from the various scatterers within the scattering volume, and the contributions to a specific frequency component in the cross spectrum will come from all points along a line of constant radial velocity; that is, one of the light solid lines perpendicular to the wind direction shown in Fig. 2. The phase difference is related to the zenith angle measured in the vertical plane parallel to the antenna baseline; that is, in the $x'-z$ plane in Fig. 2. The average phase for a given frequency component will correspond to the position at the center of the contour line of constant radial velocity; that is, along the heavy arrow representing the wind direction, if the reflectivities are symmetrically distributed or uniform. If the zenith angle measured in the vertical plane parallel to the wind direction is δ , then we can write

$$V_r = V_h \sin \delta + w. \quad (8)$$

Since

$$\sin \delta' = \sin \delta \cos(\alpha - \theta) \quad (9)$$

we get

$$V_r = V_h \frac{\sin \delta'}{\cos(\alpha - \theta)} + w. \quad (10)$$

Figure 3 illustrates the relationship between δ and δ' described by Eq. (9), and Fig. 4 of the article by Palmer

FIG. 3. Diagram showing the relationship between the inclination angles of a layer along two different vertical planes. The respective inclination angles are δ and δ' . The horizontal angle between the two vertical planes is $(\alpha - \theta)$.

et al. (1991) illustrates the relationships represented in Eqs. (8)–(10). Equation (10) has been proven more rigorously by Van Baelen (1990) by integration of the radial velocity contribution across the beam. Our derivation is not a mathematical proof, but is based on the geometry and is intended only as an illustration of the underlying principles.

The contribution to the radial velocity in Eq. (10) from the horizontal velocity component is actually the trace velocity multiplied by the sine of the zenith angle. Van Baelen (1990) was the first to point out this relationship. A number of previous studies have applied interferometer analysis to measurements made with only two receiving antennas. The assumption in those studies was that the measured radial velocity component was the projection of the horizontal velocity along the baseline; that is, $V_h \cos(\alpha - \theta)$, rather than the trace velocity along the baseline; that is, $V_h / \cos(\alpha - \theta)$. As stated in the Introduction, our own derivation proceeded in parallel with the work of Van Baelen (1990) and Van Baelen and Richmond (1990), and our original formulation was based on equations that used the velocity component rather than the trace-velocity component, similar to what was used by Kudrinski (1988), for example. The fact that the trace velocity rather than the velocity component appears in Eq. (10) is satisfying since it leads to a direct equivalence between the spaced antenna and interferometer analysis. The latter must be the case since the two methods merely provide alternative procedures for deriving the horizontal winds from the same set of measurements.

Substituting Eq. (1) in Eq. (10) and rearranging, we get

$$\phi_{ij} = \frac{kd_{ij} \cos(\alpha_{ij} - \theta)}{V_h} (V_r - w). \quad (11)$$

The phase ϕ_{ij} can be seen to vary linearly as a function of the radial Doppler velocity V_r . The line has slope m given by

$$m_{ij} = \frac{kd_{ij} \cos(\alpha_{ij} - \theta)}{V_h} \quad (12)$$

and intercept b given by

$$b_{ij} = -m_{ij}w. \quad (13)$$

Three receiving antennas yield three slopes and three intercepts. Any combination of two of the measured slopes can be used to calculate the horizontal wind components. Only the intercept from a single baseline is needed to calculate the vertical velocity component in the special case when there are no aspect sensitivity effects.

In spaced antenna analysis, the trace velocity along a given antenna baseline is calculated by dividing the antenna separation by the temporal lag at which the maximum in the cross-correlation function occurs. Trace velocities from two nonparallel antenna baselines can then be combined to yield the so-called apparent velocity. The velocities derived from the phase-difference slopes given in Eq. (12) correspond to the spaced antenna trace velocities. Indeed, Fourier transform theory predicts that a shift in the cross-correlation function will be equivalent to a phase slope in the frequency domain. Thus, the horizontal velocity vector derived by combining the various slope estimates from the interferometer analysis will be the apparent velocity. Equation (12) shows that the trace-velocity estimate will be ambiguous if the wind is perpendicular to the antenna baseline since the phase slope will be zero. In such a case, the slopes from two other baselines can be used in the analysis since the wind can only be perpendicular to one of the baselines. The same situation arises when the spaced antenna analysis is used.

From spaced antenna analysis, we know that the apparent velocity is a good estimate of the flow velocity, as long as turbulent fading within the medium is not significant. When turbulent fading is important, the apparent velocity will be an overestimate of the true velocity (e.g., see Larsen and Röttger 1989). We have not considered the effects of turbulent fading on the velocity estimates in our derivation. Such effects are known to be important in the mesosphere and stratosphere in certain height ranges (e.g., Kudrinski et al. 1989), but a number of studies have shown that the apparent velocity derived from spaced antenna measurements in the troposphere often gives a good estimate of the true flow velocity below the tropopause (e.g., Röttger and Vincent 1978; Vincent et al. 1987; and various studies summarized by Larsen and Röttger 1991). An indication of whether fading has to be considered in estimating the velocity is whether the Doppler spectra are dominated by beam broadening; that is, the spectral broadening produced by a range of radial velocity components over the finite beamwidth, or by comparable contributions from turbulent broadening and beam broadening. Simple calculations can be used to show that beam broadening will dominate in the troposphere for almost any reasonable combination of instrumental parameters. Hocking (1986) has shown the latter to be true.

If estimates of the true velocity are required, the parameters derived from the interferometer technique described here can easily be used as input for the full correlation analysis formulated by Meek (1980), for example. The parameters used in Meek's method are the lags corresponding to the maxima in the cross-correlation functions and an estimate of the characteristic fading time of the autocorrelation function. The antenna separations divided by the lags are equivalent to the trace velocities, which are also a product of the interferometer analysis. The fading time is directly related to the width of the autocorrelation function which, in turn, is related to the width of the power spectrum since the power spectrum and autocorrelation function are Fourier transform pairs. However, for the remainder of the article we will assume that the apparent velocity gives a good estimate of the true flow velocity for the reasons cited in the previous paragraph.

Using two of the measured slopes for the baselines between antennas 1 and 2 and antennas 1 and 3, for example, Eq. (12) yields

$$kd_{12}(\cos\theta \cos\alpha_{12} + \sin\theta \sin\alpha_{12}) = m_{12}V_h \quad (14a)$$

$$kd_{13}(\cos\theta \cos\alpha_{13} + \sin\theta \sin\alpha_{13}) = m_{13}V_h \quad (14b)$$

which can be rewritten as

$$X \sin\alpha_{12} + Y \cos\alpha_{12} = \frac{m_{12}}{kd_{12}} \quad (15a)$$

$$X \sin\alpha_{13} + Y \cos\alpha_{13} = \frac{m_{13}}{kd_{13}}, \quad (15b)$$

where $X = \sin\theta/V_h$ and $Y = \cos\theta/V_h$ are the inverse trace-velocity components. We can solve for X and Y to get

$$X = \frac{d_{13}m_{12} \cos\alpha_{13} - d_{12}m_{13} \cos\alpha_{12}}{kd_{12}d_{13} \sin(\alpha_{12} - \alpha_{13})} \quad (16a)$$

$$Y = \frac{d_{13}m_{12} \sin\alpha_{13} - d_{12}m_{13} \sin\alpha_{12}}{kd_{12}d_{13} \sin(\alpha_{13} - \alpha_{12})}. \quad (16b)$$

The ratio of X/Y gives the solution for θ

$$\tan\theta = -\frac{m_{12}d_{13} \cos\alpha_{13} - m_{13}d_{12} \cos\alpha_{12}}{m_{12}d_{13} \sin\alpha_{13} - m_{13}d_{12} \sin\alpha_{12}}. \quad (17)$$

The corresponding expression for V_h becomes

$$V_h = kd_{12}d_{13}[\sin(\alpha_{12} - \alpha_{13})\{[(m_{12}d_{13} \cos\alpha_{13} - m_{13}d_{12} \cos\alpha_{12})^2 + (m_{12}d_{13} \sin\alpha_{13} - m_{13}d_{12} \sin\alpha_{12})^2]^{-1/2}}. \quad (18)$$

Equation (13) gives the vertical velocity directly

$$w = -\frac{b_{ij}}{m_{ij}}. \quad (19)$$

Equation (19) comes directly from the expression for

the intercept in the linear relationship represented by Eq. (13), and the term results from setting $\phi_{ij} = 0$ in Eq. (13), which corresponds to obtaining the spectral component from the portion of the beam that points exactly in the vertical direction when there are no aspect sensitivity effects. The uncertainties in the slope estimates σ_m can be calculated by standard formulas, such as those given by Bevington (1969). We obtain

$$\sigma_m^2 = N \left[\frac{1}{N-2} \sum (\phi_n - b - mV_n)^2 \right] \times [N \sum V_n^2 - (\sum V_n)^2]^{-1} \quad (20)$$

where N is the total number of discrete values from the cross spectrum used in the line-fitting procedure and n ranges from 1 to N . The error in the intercept σ_b can be calculated from the formula (Bevington 1969)

$$\sigma_b^2 = N \left[\frac{1}{N-2} \sum (\phi_n - b - mV_n)^2 \right] \times [\sum V_n^2][N \sum V_n^2 - (\sum V_n)^2]^{-1}. \quad (21)$$

The uncertainty in the vertical velocity estimate becomes

$$\sigma_w = (b\sigma_m + m\sigma_b)m^{-2}. \quad (22)$$

Corresponding expressions for the velocity magnitude and direction error bars can be found in a straightforward way from Eqs. (17) and (18). Equation (22) shows that the error bars for w will become infinite if the slope m becomes zero. The latter can happen if the antenna baseline is perpendicular to the flow direction. The solution is to use a different pair of receiving antennas for the calculation. The method described here yields the uncertainties in a straightforward manner and with a minimum of calculations. We will discuss various aspects of implementing the scheme in practice in section 3.

b. The case with aspect sensitivity

Radar reflectivities at longer wavelengths, such as those around 6 m, are known to show aspect-sensitivity effects (e.g., Tsuda et al. 1986). The backscattered power decreases rapidly as the beam is moved away from zenith. The aspect sensitivity is not a problem in itself as long as the refractivity layers are horizontal; however, the layers are generally tilted a few degrees out of the horizontal plane. The effective beam-pointing direction then becomes essentially perpendicular to the layers since the strongest echoes come from that direction. Larsen and Röttger (1991) have discussed these problems in detail and have provided measurements of the in-beam incidence angles measured over a 4-day period. Palmer et al. (1991) have shown, based on an analysis of interferometer data, that the radial velocity measured with a nominally vertical beam is

effectively the velocity perpendicular to the refractivity layers. The recent articles discuss the effects in detail, but the problems and effects associated with inclined refractivity layers have been known for a number of years (e.g., see Röttger and Ierkic 1985).

The configuration resulting from the aspect sensitivity effects is illustrated in Fig. 4, which shows the area in the horizontal plane illuminated by the transmitted beam as the large circle. Contours of constant radial velocity are shown as the light solid lines again, and the heavy arrow that forms the diameter of the outer circle indicates both the wind direction and the mean position for the constant-radial-velocity contours when aspect sensitivity effects are unimportant. Aspect sensitivity produces enhanced backscatter for a direction perpendicular to the refractivity layers. Typically, the aspect-sensitivity pattern is much narrower than the beamwidth so that the contribution to the backscatter comes from a small area within the illuminated volume. The smaller circle represents the area within the illuminated volume that produces the largest contribution to the received signal, due to aspect sensitivity effects. If the layers are horizontal, the smaller circle will be centered at the origin, and the only effect is to make the effective beamwidth narrower than the instrumental beamwidth. However, if the layers are inclined with respect to the horizontal, the smaller circle becomes displaced from the origin, as shown in Fig. 4.

The contours of constant radial velocity enclosed within the smaller circle in Fig. 4 will be the ones that contribute to the phase measurements, and the mean position will be along the arrow that forms the diameter

of the small circle. For the particular configuration shown in Fig. 4, the phase difference corresponding to a particular value of the radial velocity will be shifted to the right; that is, along the positive x' axis as a result of the aspect-sensitivity effects. As we will show in more detail below, the slope of the phase variation as a function of the radial velocity remains the same, but the intercept of the line describing the phase variation becomes shifted. Note also that a shift in the scattering area parallel to the wind direction produces no change in the measured value; that is, displacing the center of the small circle along the direction indicated by the large arrow produces the same mean phase or location for a given value of the radial velocity as would have been measured if there were equal contributions from all points within the instrumental beamwidth. Inspection of Fig. 4 will show the latter to be true. Therefore, we can limit our derivation to consideration of the effect of the component of the off-vertical incidence angle perpendicular to the wind direction.

The radial velocity measured at any point within the scattering volume will be

$$V_r = V_h \sin \delta + w \quad (23)$$

where δ is the zenith angle measured from the center of the scattering volume along the plane defined by the horizontal wind direction and the vertical. If x'_0 and y'_0 are the coordinates of the scattering center; that is, the center of the small circle in Fig. 4, then we can write

$$\begin{aligned} \sin \delta &= \frac{[(x' - x'_0)^2 + (y' - y'_0)^2]^{1/2}}{[(x' - x'_0)^2 + (y' - y'_0)^2 + z^2]^{1/2}} \\ &\approx \frac{x' - x'_0}{z \cos(\alpha - \theta)} \end{aligned} \quad (24)$$

The approximate form of Eq. (24) was derived by using the fact that $(y' - y'_0) = (x' - x'_0) \tan(\alpha - \theta)$ and by noting that the height z is much greater than the horizontal displacements in either the x' or y' directions. Since

$$\frac{\phi}{kd} = \sin \delta' = \frac{x'}{(x'^2 + z^2)^{1/2}} \approx \frac{x'}{z} \quad (25)$$

and

$$\tan \psi_{\perp} = -\frac{x'_0}{z \sin(\alpha - \theta)} \quad (26)$$

where $\psi_{\perp}$ is the component of the incidence angle perpendicular to the wind direction, we get

$$V_r = V_h \left[\frac{\phi}{kd \cos(\alpha - \theta)} + \tan \psi_{\perp} \tan(\alpha - \theta) \right] + w. \quad (27)$$

Rewriting Eq. (27) in the form of the phase difference as a function of the radial velocity gives

FIG. 4. Diagram showing the contours of constant radial velocity (light solid lines) in a format similar to that shown in Fig. 2. Aspect-sensitivity effects localize the scatter within a small area of the larger beam, however, as shown by the circle of smaller diameter. The center of the scattering region is located at coordinates x'_0 and y'_0 .

$$\phi = \frac{kd \cos(\alpha - \theta)}{V_h} V_r - kd \tan\psi_{\perp} \sin(\alpha - \theta) - \frac{wkd \cos(\alpha - \theta)}{V_h}. \quad (28)$$

The relationship is still linear, and the slope of the line is the same as we found before without aspect-sensitivity effects. However, the intercept, given by the sum of the last two terms in Eq. (28) now depends on both the vertical velocity and the component of the off-vertical incidence angle perpendicular to the wind direction.

For a layer tilted at the angle ψ from the vertical, the tilt component along another vertical plane will be related to ψ by

$$\tan\psi' = \tan\psi \cos\beta \quad (29)$$

where ψ' is the tilt angle in the vertical plane rotated by angle β in azimuth with respect to the vertical plane in which ψ is measured. We can then express $\tan\psi_{\perp}$ as

$$\tan\psi_{\perp} = \tan\psi \cos\left(\frac{\pi}{2} + \theta - \chi\right) \quad (30)$$

where χ is the azimuth along which the layer is tilted. Finally, we end up with

$$\phi = \frac{kd \cos(\alpha - \theta)}{V_h} V_r + kd \tan\psi \sin(\theta - \chi) \times \sin(\alpha - \theta) - \frac{wkd \cos(\alpha - \theta)}{V_h}. \quad (31)$$

For each baseline, there are two fitted parameters. The slope of the fitted line is given by

$$m_{ij} = \frac{kd_{ij} \cos(\alpha_{ij} - \theta)}{V_h}. \quad (32)$$

The intercept is given by

$$b_{ij} = kd_{ij} \tan\psi \sin(\theta - \chi) \sin(\alpha_{ij} - \theta) - m_{ij}w. \quad (33)$$

A more useful form of Eq. (33) is

$$b_{ij} = kd_{ij}(\tan\psi_x \cos\theta + \tan\psi_y \sin\theta) \times \sin(\alpha_{ij} - \theta) - m_{ij}w. \quad (34)$$

Any pair of the slope estimates can be used to calculate the horizontal wind components from Eq. (19). The slopes and intercepts can then be used to calculate ψ_x , ψ_y , and w from Eq. (34). The error estimation can be carried out as described in section 2b.

Palmer et al. (1991) presented some examples of the tilt-angle values derived by this technique. The half-

FIG. 5. (a) Coherence function amplitude for a height with a broad spectrum. (b) Phase of the coherence function. (c) Determination of the window for fitting the phase based on the implied horizontal velocity determined from the phase of the coherence function. (d) Fitted line and actual phase values.

hour average values were found to have magnitudes of $\sim 0.5^\circ$, which agrees, in general, with the results of Röttger et al. (1990) and Larsen and Röttger (1991), who derived the tilt angles by a different method. Röttger et al. (1990) have also presented results of tilt angle measurements based on data from the Chung-Li radar in Taiwan, which agreed with the other measurements as far as overall average magnitude is concerned.

An advantage of the analysis technique presented here over those that use Doppler measurements of the vertical velocity is that it leads to estimates of the true vertical velocity. Off-vertical in-beam incidence angles result in measured radial velocities that have a component that is the projection of the horizontal velocity along the line of sight. Although, the angles are typically small, the difference between the true vertical velocity and the radial velocity in a nominally vertical beam can be significant, as shown by Röttger and Ierkic (1985), Larsen and Röttger (1991), and Palmer et al. (1991).

3. An example of the application of the method

Our goal is not to make an exhaustive test of the technique or to compare results provided by different types of analysis. The purpose of the paper is merely

to provide a framework for carrying out the analysis of interferometer data, including the complexities imposed by aspect-sensitivity effects; however, the easiest way to illustrate the method is to apply it to a small sample of data. The observations were made on 24 October 1989 from 2100 to 2400 LST with the MU radar located in Shigaraki, Japan. The great flexibility of the radar system makes it possible to operate the radar in a number of different modes. In this case, the central portion of the array was used for transmission, and three spatially separated receiving antennas were used. The dataset has been described in more detail by Palmer et al. (1990, 1991).

The interferometer analysis, as described in the previous section, was carried out, including the calculation of the complex coherence function. Examples of the calculated functions from two heights are shown in Figs. 5a–d and Figs. 6a–d. Figure 5 corresponds to the case of a broad Doppler spectrum. Figure 5a shows that the spectral peak can be discerned between the approximate limits of $\pm 5.0 \text{ m s}^{-1}$. Over the same velocity or frequency interval, the phase in Fig. 5b can be seen to be varying linearly, as expected for small zenith angles. Actually, the phase changes by more than 2π rad, which produces the two discontinuous jumps near $\pm 3.5 \text{ m s}^{-1}$. As illustrated in Fig. 5a, the phase

FIG. 6. (a) Coherence function amplitude for a height with a narrow spectrum. (b) Corresponding phase for the coherence function. (c) Window for the fitting determined from the beam broadening effects due to the approximate horizontal velocity. (d) Actual phase values and fitted line.

of the coherence function varies linearly only in the part of the Nyquist spectral interval that contains signal. Therefore, a window based on the beam broadening caused by the horizontal wind was chosen. Since the phase information defines a pointing direction, the velocities in the Doppler spectrum can be interpreted as the projection of the horizontal velocity along those pointing directions. Figure 5c shows the radial velocities converted to approximate horizontal velocities. The velocities should lie along a horizontal line if the winds are uniform. The median of the approximate horizontal velocity estimates was calculated and used to find the beam-broadened spectral width, which was used to define the window for the linear fitting procedure. The line fitted to the phase variation is shown in Fig. 5d superimposed on the actual phase values. Note that the horizontal axis in Fig. 5d has a different scale than the axis in Figs. 5a–c. More work is needed to refine the procedure for estimating the spectral window for fitting the line, although this procedure produced ad-

equate results for the case described here, in which the wind was almost entirely parallel to one of the antenna baselines.

Figure 6 shows an example involving a narrow spectral peak. The window for fitting is again determined by the beam broadening due to the approximate horizontal velocity as shown in Fig. 6c. The fitted line and the phase values are shown in Fig. 6d.

The winds were calculated for heights between 6 and 16 km based on 30-min-average spectra such as those shown in Figs. 5 and 6. The results are presented in Fig. 7a. The true vertical velocity was also calculated as indicated. The Doppler velocity measured with a vertical beam is effectively the radial velocity perpendicular to the tilted refractivity layers, as discussed by Röttger and Ierkic (1985), Larsen and Röttger (1991), and Palmer et al. (1991). The resulting velocity estimates are then contaminated by a component of the horizontal wind. However, the interferometer analysis described here yields the true vertical velocity, in ad-

FIG. 7. On the left, profiles of the zonal and meridional wind determined by the interferometer method for the data from 2145 to 2215 LST. The true vertical velocity derived by the interferometer method is shown on the right.

dition to the layer tilt angles. The latter are not shown here but were presented by Palmer et al. (1991).

The curves for both the horizontal and vertical velocities in Figs. 7a and 7b have error bars plotted, although they are more obvious in the panel on the right. The uncertainty in the horizontal velocity determination is $\sim 0.5 \text{ m s}^{-1}$ at most heights. The uncertainties in the vertical velocity estimates are larger in comparison to the magnitude of the velocities than is the case for the horizontal winds. Typical magnitudes of the uncertainties of the vertical velocities at most heights are in the range from 5 to 10 cm s^{-1} .

For comparison, the horizontal winds deduced by a Doppler method are shown in Fig. 8a. The Doppler estimates lie outside the error bars derived from the interferometer analysis at many heights, indicating that the differences are attributable to more than statistical uncertainty, although there is no way to determine

which profile is a better estimate of the true winds with the data available here. The data that produced the curves in Fig. 7 were obtained between 2145 and 2215 LST, and the data in Fig. 8 were obtained between 2110 and 2120 LST, since the sampling scheme was different. Therefore, the differences in the profiles may be due to temporal evolution of the flow. However, the differences in the vertical velocity estimates obtained during the two observation intervals can be explained in large part by the effect of the off-vertical incidence angles that affected the Doppler measurements, as shown by the detailed analysis carried out in the study by Palmer et al. (1991), indicating that the temporal evolution, at least in the vertical velocities, was small during the gap in the observations.

The vertical velocities deduced by the Doppler method are shown in Fig. 8b. There are significant differences between the profiles in Figs. 7b and 8b for the

FIG. 8. On the left, profiles of the zonal and meridional velocity derived by the Doppler method for the data from 2110 to 2120 LST. The vertical velocity profile derived by the Doppler method is shown on the right.

reasons related to tilted refractivity layers already cited. Although the uncertainties in the vertical velocity estimates are relatively large in comparison to the velocity magnitudes, the differences between the vertical velocities derived by the interferometer and Doppler methods are larger than the uncertainties over a large part of the height range.

Figure 8 shows a gap in the vertical-velocity profile determined from the Doppler method. The received power in the vertical and off-vertical beams for the Doppler data near 14 km was comparable. However, the vertical spectra did not have well-defined peaks, and the spectral fitting procedure did not converge. The peaks in the off-vertical spectra were better defined, and horizontal wind estimates were possible. The interferometer profiles in the same height range show an increase in the size of the error bars, although both the horizontal and vertical velocities could be determined.

4. Discussion

An advantage of the analysis method described here is that the number of computations required to implement it are reduced in comparison to the spaced antenna analysis, for example. The spaced antenna analysis can provide the same parameters as the interferometer method, namely, the horizontal and true vertical velocities, and the layer tilt angles (Larsen and Röttger 1989, 1991); however, more calculations are required. The horizontal winds are calculated from the temporal lags of the peaks in the auto- and cross-correlation analysis. The tilt angle is calculated from the phase of the cross correlation at zero lag. The radial velocity in the vertical beam is derived from the variation of the phase of the autocorrelation function, and the corrected true vertical velocity is calculated by combining the vertical-beam radial velocity, tilt angles, and horizontal wind information. Error-bar estimation requires an iterative procedure. The interferometer method as outlined here only requires that the coherence function be calculated for each antenna pair and that a line be fitted to the phase variation. The slope and intercept parameters then yield all the same physical quantities as the spaced antenna analysis. There is a minimum of computational complexity, and simple expressions for the uncertainties in the calculated quantities emerge from the derivation.

Not only do the calculations appear to be simple enough to carry out in real time, but the implementation of the hardware is also straightforward since all beams point vertically. The Doppler method requires beam pointing in a number of directions, which adds some complexity to the hardware and leads to assumptions about uniformity of the wind field over the sometimes large horizontal separations between beam positions. The vertically pointing beam setup used for the interferometer measurements requires no beam steering. However, three receivers are necessary. Many

of the same advantages and disadvantages of interferometry apply to spaced antenna systems, which have the same hardware requirements (Larsen and Röttger 1989).

Ideally, the coherence values from all antenna pairs will be usable for the analysis, but in practice, the wind may be parallel to one of the baselines so that the coherence is low along the other two baselines. However, since the wind magnitude and direction are calculated as a first step by using the fitted slopes, such special situations should be evident before the second step of finding the in-beam incidence angles and true vertical velocities is implemented using the fitted intercept values. Clearly, more study of such potential practical problems is needed.

5. Conclusion

Although the basic idea of using the linear variation of the phase in interferometer measurements to infer the horizontal winds has been discussed by a number of authors, no complete analysis method has been proposed. Also, the additional complexity introduced by aspect-sensitivity effects has not been considered fully in the past in connection with the interferometer analysis. The analysis method introduced here involves only simple calculations but yields all the parameters that are typically of interest for operational or research wind profiler applications. Clearly, more detailed studies are needed in the future to determine how the interferometer method compares to the spaced antenna or Doppler beam swinging method in terms of both ease of implementation and accuracy and precision of the results.

Acknowledgments. We want to thank Dr. Joel Van Baelen who pointed out an important error in the original version of our manuscript. We also thank him for his input during many useful and interesting discussions about interferometry in general and our parallel efforts in particular over the last few months. We also thank Dr. Erhan Kudeki and Dr. Steve Franke who had many good insights about the work and the interferometry technique. MFL received support from the Ministry of Education, Research, and Culture of Japan and from AFOSR Contract F49620-88-C-0121 during the course of the study. RDP was supported as a Japan Society for the Promotion of Science (JSPS) postdoctoral fellow. The MU Radar is owned and operated by the Radio Atmospheric Science Center of Kyoto University.

REFERENCES

- Adams, G. W., J. W. Brosnahan, D. C. Walden, and S. F. Nerney, 1986: Mesospheric observations using a 2.66-MHz radar as an imaging Doppler interferometer: Description and first results. *J. Geophys. Res.*, **91**, 1671–1683.

- Bevington, P. R., 1969: *Data Reduction and Error Analysis for the Physical Sciences*, McGraw-Hill, 336 pp.
- Briggs, B. H., 1980: Radar observations of atmospheric winds and turbulence: A comparison of techniques. *J. Atmos. Terr. Phys.*, **42**, 823–833.
- Doviak, R. J., and D. S. Zrnić, 1984: *Doppler Radar and Weather Observations*. Academic Press, 458 pp.
- Farley, D. T., H. M. Ierkeć, and B. G. Fejer, 1981: A new technique for studying plasma turbulence in the ionosphere. *J. Geophys. Res.*, **86**, 1467–1472.
- Hocking, W. K., 1986: Observation and measurement of turbulence in the middle atmosphere with a VHF radar. *J. Atmos. Terr. Phys.*, **48**, 655–670.
- Kudeki, E., 1988: Radar interferometer observations of mesospheric echoing layers at Jicamarca. *J. Geophys. Res.*, **93**, 5413–5421.
- , F. Surucu, and R. F. Woodman, 1990: Mesospheric wind and aspect sensitivity measurements at Jicamarca using radar interferometry and post-statistics steering techniques. *Radio Sci.*, **25**, 595–612.
- Larsen, M. F., and J. Röttger, 1989: The spaced antenna technique for radar wind profiling. *J. Atmos. Oceanic Technol.*, **6**, 920–938.
- , and —, 1991: VHF radar measurements of in-beam incidence angles and associated vertical-beam radial velocity corrections. *J. Atmos. Oceanic Technol.*, **8**, 477–490.
- Liu, C.-H., J. Röttger, C. J. Pan, and S. J. Franke, 1990: A model for spaced antenna observational mode for MST radars. *Radio Sci.*, **25**, 551–563.
- Meek, C. E., 1980: An efficient method for analysing ionospheric drifts data. *J. Atmos. Terr. Phys.*, **42**, 835–839.
- Palmer, R. D., R. F. Woodman, S. Fukao, T. Tsuda, and S. Kato, 1990: Three-antenna post-statistic beam steering using the MU radar. *Radio Sci.*, **25**, 1105–1110.
- , M. F. Larsen, R. F. Woodman, S. Fukao, M. Yamamoto, T. Tsuda, and S. Kato, 1991: VHF radar interferometry measurements of vertical velocity and the effect of tilted refractivity surfaces on standard Doppler measurements. *Radio Sci.*, **26**, 417–427.
- Röttger, J., and R. A. Vincent, 1978: VHF radar studies of tropospheric velocities and irregularities using spaced antenna techniques. *Geophys. Res. Lett.*, **5**, 917–920.
- , and H. M. Ierkeć, 1985: Postset beam steering and interferometer applications of VHF radars to study winds, waves, and turbulence in the lower and middle atmosphere. *Radio Sci.*, **20**, 1461–1480.
- , C.-H. Liu, J. K. Chao, A. J. Chen, C. J. Pan, and I.-J. Fu, 1990: Spatial interferometer measurements with the Chung-Li VHF radar. *Radio Sci.*, **25**, 503–515.
- Tsuda, T., T. Sato, K. Hirose, S. Fukao, and S. Kato, 1986: MU radar observations of the aspect sensitivity of backscattered VHF echo power in the troposphere and lower stratosphere. *Radio Sci.*, **21**, 971–980.
- Van Baelen, J. S., 1990: Comparison of clear air atmospheric radar techniques for the study of atmospheric dynamics in the troposphere and the stratosphere. NCAR Cooperative Thesis No. 128, 191 pp. [Available from NCAR, Boulder, CO.]
- , and A. D. Richmond, 1990: Radar interferometry technique: 3-D wind measurement theory. *Radio Sci.*, (submitted.)
- Vincent, R. A., P. T. May, W. K. Hocking, W. G. Elford, B. H. Candy, and B. H. Briggs, 1987: First results with the Adelaide VHF radar: Spaced antenna studies of tropospheric winds. *J. Atmos. Terr. Phys.*, **49**, 353–366.